松　山　大　学　論　集
第 32 巻 第 5 号 抜 刷
２０２０ 年 12 月 発 行

Basic Research Spending, Applied Research Subsidy, and Growth Cycles

Kunihiko Konishi

Basic Research Spending, Applied Research Subsidy, and Growth Cycles

Kunihiko Konishi*

abstract

This study constructs a variety-expansion growth model that integrates basic research to analytically examine growth cycles. We show that the equilibrium path can exhibit two-period cycles through the interplay between applied and basic research. In addition, we explore the effects of change in basic research spending and applied research subsidy. Under certain conditions, the steady-state growth rate increases when basic research spending or applied research subsidy increases. However, the effects on the possibility of cyclical instability differ by applied and basic research policies. An increase in basic research spending reduces the possibility of cyclical instability, while an increase in applied research subsidy raises the possibility of cyclical instability.

1. Introduction

It is widely known that the long-run growth of developed countries is fluctuating, and a number of studies have investigated the existence of endogenous cycles from various perspectives using R&D-based models.[1)] However, few studies focus on the linkage between basic research and economic fluctuation. Empirical

---

* Faculty of Economics, Matsuyama University, 4-2 Bunkyo-cho, Matsuyama-shi, Ehime 790-8578, JAPAN

1) See Shleifer (1986), Gale (1996), Deissenberg and Nyssen (1998), Francois and Shi (1999), Freeman et al. (1999), Matsuyama (1999), Francois and Lloyd-Ellis (2003, 2008, 2009), and Wälde (2005).

studies argue that basic research contributes to the development of the economy (Griliches, 1986; Jaffe, 1989; Mansfield, 1998; Cohen et al., 2002) and the theoretical implications of basic research policy have been considered through various macroeconomic viewpoints.[2] Thus, it is important to consider the existence of endogenous cycles within a R&D-based model that integrates basic research.

The present study theoretically examines the role of basic research in growth cycles. To do this, we incorporate basic research into a variety-expansion model following Grossman and Helpman (1991). In our model, basic research generates ideas, whereas applied research commercializes them by transforming them into blueprints for new varieties of consumption goods. These two research sectors interplay through knowledge spillovers. Further, we assume that basic research is publicly funded－and thus, that the government can control the level of basic research. According to Table 2 in Gersbach et al. (2013), which summarizes 2009 data from a selection of 15 countries, the average share of basic research that was financed by governments and higher educational institutions was 77.39%; that is, basic research is mainly funded by the government and conducted at universities or other public research institutions. In addition, on an average, 76.62% of applied research was financed by business enterprises and private non-profit institutions; that is, applied research is primarily performed by private firms motivated by their own benefits.

We show that the equilibrium path can exhibit two-period cycles through the interplay between applied and basic research. The key driving force that gives rise to cycles is the knowledge spillovers between applied and basic research. In our

2) Cozzi and Galli (2009, 2013, 2014), Chu et al. (2012), and Chu and Furukawa (2013) analyze the profit-division rules between applied and basic research. Park (1998), Gersbach et al. (2018), and Konishi (2018) consider the policy implications of basic research spending financed by the government. Gersbach et al. (2013) and Gersbach and Schneider (2015) examine the interaction between investment in basic research and open-economy issues.

model, because there is a one-to-one relationship between ideas and potential blueprints, we assume that the applied research sector's knowledge spillover from basic research is derived from the ideas awaiting commercialization. If commercialization in the applied research sector is much larger than the creation of ideas in the basic research sector, the applied research sector's knowledge spillover from basic research decreases in the next period. This decreases the growth in the number of differentiated goods. Further, larger commercialization in the applied research sector increases the basic research sector's knowledge spillover from applied research, and consequently, there is increased growth in the number of ideas that have been generated through basic research. Owing to these interactions, the presence of knowledge spillovers between applied and basic research can lead to perpetual economic fluctuation.

Furthermore, we investigate the effects of change in basic research spending and applied research subsidy. Under some assumptions, the steady-state growth rate increases when basic research spending or applied research subsidy increases. However, these two policies differ in their effects on the possibility of cyclical instability. An increase in basic research spending reduces the possibility of cyclical instability, while an increase in applied research subsidy raises the possibility of cyclical instability.

The present study is closely related to Growiec and Schumacher (2013). They build an R&D-based model where *radical innovation* (viewed as basic research) creates technological opportunity and *incremental innovation* (viewed as applied research) raises the differentiated goods but reduces technological opportunity. In this set-up, they obtain the possibility of oscillatory dynamics for a large variety of parameter values. However, they do not investigate the effects of basic research spending and applied research subsidy. This study is also related to Li (2001), who presents a model in which *technological research* (viewed as applied research) increases the differentiated goods and *scientific research* (viewed as basic research) accelerates technological progress. In his model, growth cycles occur because of the assumption that scientific breakthroughs arrive in discrete jumps. Therefore, the effects of change in government policy on the possibility of cyclical instability cannot be analyzed.

Other related studies (Aloi and Lasselle, 2007; Haruyama, 2009; Li and Zhang, 2014) examine the effects of applied research subsidy on the possibility of cyclical instability. Haruyama (2009) shows that cycles can arise in the standard R&D-based model of Grossman and Helpman (1991) and that optimal applied research subsidy fails to eliminate cycles. Aloi and Lasselle (2007) and Li and Zhang (2014) show that applied research subsidy can stabilize innovation cycles and increase welfare using the Matsuyama (1999) model. However, these studies do not consider the role of basic research.

The rest of this paper is organized as follows. Section 2 establishes the model used in this study. Section 3 derives the equilibrium dynamics of the economy and analyzes how the policy affects the steady-state growth rate and the possibility of cyclical instability. Section 4 presents the numerical examples. Finally, Section 5 concludes the paper.

2. Model

We apply Grossman and Helpman's (1991) concept of variety expansion to a two-period overlapping-generations model following Diamond (1965). An individual lives for two periods. A cohort born in period t is called generation t. Therefore, there exist two generations in period t; that is, generation t (the young generation) and generation $t-1$ (the old generation). In each period, the size of the newly born cohort is given by one. Each individual supplies one unit of skilled labor and L units of unskilled labor inelastically in their young period and retires in the old period. The factor market is perfectly competitive, and the goods market is monopolistically competitive, as explained below. Individuals have perfect foresight.

2.1. Consumers

Each consumer born at period t maximizes utility,

$$U_t = \log C_{1,t} + \rho \log C_{2,t+1}$$

where $\rho \in (0, 1)$ is the subjective discount factor, $C_{1,t}$ is the consumption when young, and $C_{2,t+1}$ is the consumption when old. The budget constraint is as follows :

$$E_{1,t} + S_t = (1-\tau_t)(w_t^s + w_t^u L) \quad and \quad E_{2,t+1} = (1+r_{t+1}) S_t,$$

where $E_{1,t}$ and $E_{2,t+1}$ represent the consumption expenditures of the young as well as the old agents of generation t at times t and $t+1$, respectively, and S_t is the saving in youth. Let w_t^s, w_t^u, and r_{t+1} be the wage rate for skilled labor, the wage rate for unskilled labor, and the interest rate. We assume that in each period, the government runs a balanced budget, where it finances its spending with taxes τ_t levied on the wage incomes of the young generation, as explained below. We specify the subutility function $C_{i,t}$ $(i = 1, 2)$ as

$$C_{i,t} = \left[\int_0^{A_t} C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1}$$

where $C_{i,t}(j)$ is the consumption of good j of the young or old agent at time t and A_t is the number of differentiated goods. We assume that $\varepsilon > 1$. ε denotes the elasticity of substitution between any two products. By maximizing the subutility function $C_{i,t}$ subject to the budget constraint $E_{i,t} = \int_0^{A_t} p_t(j) c_{i,t}(j) dj$, we obtain the demand function for good j as follows :

$$c_{i,t}(j)=\frac{p_t(j)^{-\varepsilon}E_{i,t}}{\int_0^{A_t}p_t(k)^{1-\varepsilon}dk}, \tag{2}$$

where $p_t(j)$ is the price of good j. Substituting demand function (2) into (1) yields

$$C_{i,t}=\frac{E_{i,t}}{P_{D,t}},$$

where $P_{D,t}$ is the price index defined as

$$P_{D,t}=\left[\int_0^{A_t}p_t(j)^{1-\varepsilon}dj\right]^{\frac{1}{1-\varepsilon}}.$$

By solving the intertemporal utility maximization, the saving function of each consumer becomes

$$S_t=(1-\tau_t)\frac{\rho}{\rho+1}(w_t^s+w_t^u L). \tag{3}$$

In addition, the total demand for good j can be given by

$$c_t(j)=\frac{p_t(j)^{-\varepsilon}E_t}{\int_0^{A_t}p_t(k)^{1-\varepsilon}dk}, \tag{4}$$

where $E_t\equiv E_{1,t}+E_{2,t}$ is the total expenditure at time t. Following Grossman and Helpman (1991), we normalize the total expenditure at unity, and thus, $E_t=1$.

2.2. Production

We assume that each differentiated good that has been created by applied research is produced by a single firm because the good is infinitely protected by a patent. We further assume that the production function of good j is the following

Cobb-Douglas form :

$$x_t(j) = a_x [l_t^s(j)]^{\theta} [l_t^u(j)]^{1-\theta}, \; a_x > 0 \quad and \quad \theta \in (0, 1),$$

where $x_t(j)$ is the output of good j, a_x is the productivity of production, θ is the intensity of skilled labor in production, and $l_t^s(j)$ and $l_t^u(j)$ denote the amount of skilled and unskilled labor devoted to producing good j. From cost minimization, the unit cost function $z(w_t^s, w_t^u)$ is

$$z(w_t^s, w_t^u) = a_x^{-1} \theta^{-\theta} (1-\theta)^{\theta-1} (w_t^s)^{\theta} (w_t^u)^{1-\theta}, \tag{5}$$

Applying Shephard's lemma, we obtain demand functions for skilled and unskilled labor as follows :

$$l_t^s(j) = \theta \frac{z(w_t^s, w_t^u)}{w_t^s} x_t(j), \tag{6}$$

$$l_t^u(j) = (1-\theta) \frac{z(w_t^s, w_t^u)}{w_t^u} x_t(j). \tag{7}$$

The firm manufacturing good j (firm j) maximizes its profit :

$$\pi_t(j) = p_t(j) x_t(j) - z(w_t^s, w_t^u) x_t(j).$$

Then, firm j charges the following price :

$$p_t(j) = p_t = \frac{\varepsilon}{\varepsilon - 1} z(w_t^s, w_t^u). \tag{8}$$

Therefore, all goods are priced equally. Pricing rules (8) and (4) yield

$$x_t(j) = x_t = \frac{\varepsilon - 1}{\varepsilon} \frac{1}{z(w_t^s, w_t^u) A_t}. \tag{9}$$

Then, the monopoly profits of a firm are as follows :

$$\pi_t(j) = \pi_t = \frac{1}{\varepsilon A_t}.$$

2.3. Basic and applied research

Following Chu and Furukawa (2013) and Gersbach et al. (2018), we assume that basic research generates ideas, whereas applied research commercializes them by transforming them into blueprints for new differentiated goods. Each research activity requires skilled labor input. We assume the following basic research technology :

$$B_{t+1} - B_t = a_B F_B(A_t, B_t) L_{B,t}, \tag{10}$$

where B_t, $L_{B,t}$, a_B, and $F_B(A_t, B_t)$ represent the measure of ideas that have been generated through basic research, the amount of skilled labor devoted to basic research, the productivity of basic research, and the knowledge spillover function in the basic research sector, respectively. Basic research productivity depends on both basic and applied research. As discussed in Gersbach et al. (2018), basic researchers benefit from applied research (e.g., discovering unresolved research problems, disclosing potentially new areas of science, and applying novel instrumentation and methodologies). Therefore, the productivity of basic research increases when applied research progresses.

Next, we consider applied research activities. Applied researchers commercialize the ideas generated by basic research. Further, the commercialization turns ideas into blueprints for new differentiated goods ; that is, A_t increases. Denote $W_t \equiv B_t - A_t$ as the pool of ideas awaiting commercialization. When an applied researcher invests $\dfrac{q_t}{a_A F_A(A_t, W_t)}$ units of skilled labor, he/she can commercialize the idea j with probability q_t. Since time is discrete, duplication, that is, different applied researchers simultaneously succeeding in innovation, is possible. To avoid this complication, we assume a single innovator that engages in the commercialization of each idea. In this set-up, the aggregate level of expansion in the differentiated goods is as follows:

$$A_{t+1} - A_t = q_t W_t = a_A F_A(A_t, W_t) L_{A,t}, \tag{11}$$

where $L_{A,t}$, a_A, and $F_A(A_t, W_t)$ represent the amount of skilled labor devoted to applied research, the productivity of applied research, and the knowledge spillover function in the applied research sector, respectively. Note that the applied research sector's knowledge spillover from basic research is not B_t but W_t. In this study, the knowledge generated by basic research is divided into two parts: the commercialized ideas and the ideas awaiting commercialization. Because there is a one-to-one relationship between ideas and potential blueprints, the knowledge derived from the commercialized ideas overlaps with the knowledge derived from the applied research, A_t. Hence, we assume that the applied research sector's knowledge spillover from basic research is derived from the ideas awaiting commercialization, W_t.

The applied research sector is assumed to be competitive, and the free entry condition is as follows:

$$v_t = \frac{(1 - s_A) w_t^s}{a_A F_A(A_t, W_t)} \quad \textit{if} \quad A_{t+1} - A_t > 0, \tag{12}$$

where $s_A \in [0, 1]$ is subsidy for applied research. We assume that the subsidy rate s_A is held constant over time. Next, we consider a no-arbitrage condition. The expected returns on the stocks equate to the risk-free interest in the financial market. The shareholders of the stocks earn dividends π_{t+1} and capital gains $v_{t+1} - v_t$. We assume that the shareholders hold a well-diversified portfolio of shares of innovators. Under this assumption, the shareholders can earn a safe return by holding this portfolio, because the risks involved with any particular innovator are idiosyncratic. Therefore, we obtain the following no-arbitrage condition :

$$r_{t+1}v_t = \pi_{t+1} + v_{t+1} - v_t.$$

2. 4. Government

Basic research and subsidy for applied research are financed by the wage income taxes of the young generation, and the government runs a balanced budget in each period. That is, the government budget constraint becomes

$$\tau_t (w_t^s + w_t^u L) = w_t^s L_{B,t} + s_A w_t^s L_{A,t}. \tag{13}$$

For simplicity, we assume that the government keeps the number of public researchers constant (i. e., $L_{B,t} = L_B$) and that the tax rate τ_t is determined to satisfy the government budget constraint.

2. 5. Market-clearing condition

We consider the labor-market conditions. Skilled labor is used for production, applied research, and the employment of public researchers. The market-clearing condition for skilled labor becomes

$$A_t l_t^s + L_{A,t} + L_B = 1. \tag{14}$$

The market-clearing condition for unskilled labor is

$$A_t l_t^u = L. \tag{15}$$

Finally, we consider the equilibrium condition of the financial markets. The total savings of young agents in period t must be used for the investment or for the purchase of the existing stocks. Hence, the saving function (3) implies the following asset market equilibrium condition:

$$(1-\tau_t)\frac{\rho}{\rho+1}(w_t^s+w_t^u L)=(1-s_A)w_t^s L_{A,t}+A_t v_t. \tag{16}$$

3. Equilibrium

3.1. Dynamic system

We characterize the equilibrium paths in this economy. For analytical tractability, the spillover functions $F_A(A_t, W_t)$ and $F_B(A_t, B_t)$ are assumed to have the Cobb-Douglas forms as follows:

$$F_A(A_t, W_t)=A_t^{\alpha}W_t^{1-\alpha} \quad and \quad F_B(A_t, B_t)=A_t^{\beta}B_t^{1-\beta}.$$

By using (7) and (9), the market-clearing condition for unskilled labor (15) becomes

$$w_t^u = w^u = \frac{(1-\theta)(\varepsilon-1)}{\varepsilon L}. \tag{17}$$

In addition, (6) and (9) yield

$$A_t l_t^s = \theta\frac{\varepsilon-1}{\varepsilon}\frac{1}{w_t^s}. \tag{18}$$

By using (12), (13), (14), (16), (17), and (18), we obtain

$$\frac{\varepsilon-1}{\varepsilon}\frac{1}{w_t^s}=\frac{(1-s_A)(\rho+1)}{\rho+(1-s_A)\theta}\left[\frac{1-L_B}{\rho+1}+\frac{1}{a_A}\left(\frac{\lambda_t}{1-\lambda_t}\right)^{1-\alpha}\right], \tag{19}$$

where $\lambda_t \equiv \frac{A_t}{B_t}$. From (14), (18), and (19), the amount of skilled labor devoted to applied research is as follows:

$$L_{A,t}=\frac{1}{\rho+(1-s_A)\theta}\left[\rho(1-L_B)-\frac{(1-s_A)(\rho+1)\theta}{a_A}\left(\frac{\lambda_t}{1-\lambda_t}\right)^{1-\alpha}\right]. \quad (20)$$

From (14), the growth in the differentiated goods is as follows :

$$g_t^A\equiv\frac{A_{t+1}-A_t}{A_t}=a_A\left(\frac{1-\lambda_t}{\lambda_t}\right)^{1-\alpha}L_{A,t}. \quad (21)$$

Similarly, (10) yields

$$g_t^B\equiv\frac{B_{t+1}-B_t}{B_t}=a_BL_B\lambda_t^{\beta}. \quad (22)$$

Note that the entity enclosed in the curly brackets of (20) is non-positive if the applied research is not undertaken. Hence, the condition that the applied research is not conducted is as follows :

$$\left(\frac{1-\lambda_t}{\lambda_t}\right)^{1-\alpha}\leq\frac{(1-s_A)(\rho+1)\theta}{\rho a_A(1-L_B)} \quad (23)$$

Let us define $\hat{\lambda}$ by (23) when it holds with equality. Since the left-hand side of (23) is decreasing in λ_t, $\lambda_t\geq\hat{\lambda}$ implies that $L_{A,t}=0$. Therefore, by using (20), (21), (22), and the definition of λ_t, the dynamics of λ_t are expressed as

$$\lambda_{t+1}=\begin{cases}\dfrac{\rho a_A(1-L_B)\lambda_t^{\alpha}(1-\lambda_t)^{1-\alpha}+\rho[1-(1-s_A)\theta]\lambda_t}{[\rho+(1-s_A)\theta](1+a_BL_B\lambda_t^{\beta})}\equiv\Phi(\lambda_t) & if \quad \lambda_t<\hat{\lambda}, \\ \dfrac{\lambda_t}{1+a_BL_B\lambda_t^{\beta}}\equiv\Psi(\lambda_t) & if \quad \lambda_t\geq\hat{\lambda}.\end{cases} \quad (24)$$

The equilibrium dynamics of this economy can be described by λ_t.

3.2. Steady state and Equilibrium path

In this subsection, we investigate the steady state and the equilibrium path. Asterisks represent variables in the steady state. Under the Cobb-Douglas specifications of $F_A(A_t, W_t)$ and $K(A_t, B_t)$, $\lambda^* = 0$ can be the steady state. However, we show in Appendix A that $\lambda^* = 0$ is unstable. We then consider the non-trivial steady state, which is determined as $\lambda_{t+1} = \lambda_t = \lambda^* > 0$. In the non-trivial steady state, $\lambda^* = \Phi(\lambda^*)$ holds because the definition of λ_t and (22) imply that $g_t^A = g_t^B = g^*$ and $g^* > 0$ as long as $L_B > 0$. Therefore, from (24), the equation that determines the steady-state value is as follows:

$$[\rho+(1-s_A)\theta][1+a_B L_B(\lambda^*)^{\beta}] = \rho a_A(1-L_B)\left(\frac{1-\lambda_t}{\lambda_t}\right)^{1-\alpha} + \rho[1-(1-s_A)\theta]. \tag{25}$$

We define the left-hand side of (25) as $X_L(\lambda^*)$ and the right-hand side of (25) as $X_R(\lambda^*)$. The property of $X_L(\lambda^*)$ and $X_R(\lambda^*)$ is as follows:

$$X_L'(\lambda^*) = \beta[\rho+(1-s_A)\theta]a_B L_B(\lambda^*)^{\beta-1} > 0,$$
$$X_R'(\lambda^*) = -(1-\alpha)\rho a_A(1-L_B)(1-\lambda^*)^{-\alpha}(\lambda^*)^{\beta-1} < 0,$$
$$\lim_{\lambda^*\to 0} X_L(\lambda^*) < \lim_{\lambda^*\to 0} X_R(\lambda^*),$$
$$\lim_{\lambda^*\to 1} X_L(\lambda^*) > \lim_{\lambda^*\to 1} X_R(\lambda^*).$$

Thus, we confirm that there is a unique non-trivial steady state.

Next, we examine the dynamic system (24). As shown in Appendix A, we obtain the following lemma:

Lemma 1. *$\Phi(\lambda)$ is unimodal if $0 < L_B \leq 0.5$, $0.05 \leq \alpha \leq 0.9$, $\alpha \geq \beta$, $\alpha+\beta \leq 1$, and $a_A \geq a_B$.*

In this study, because of the one-to-one relationship between ideas and potential blueprints, $0 < \lambda_t < 1$ certainly holds. Therefore, to ensure that $0 < \lambda_t < 1$, we impose $\Phi(\tilde{\lambda}) < 1$, where $\tilde{\lambda}$ represents the value that maximizes $\Phi(\lambda_t)$, that is, $\Phi'(\tilde{\lambda}) = 0$ holds. With regard to the local stability, we obtain the following proposition (see Appendix B for details):

Proposition 1. *Let us define*

$$\Gamma(\lambda^*) \equiv \frac{2\rho+(1-s_A)\theta(1-\rho)}{1-L_B} + \frac{\rho a_A(\alpha-\lambda^*)}{(\lambda^*)^{1-\alpha}(1-\lambda^*)^{\alpha}} + \frac{(1-\beta)[\rho+(1-s_A)\theta]a_B L_B(\lambda^*)^{\beta}}{1-L_B}.$$

1. *If* $\Gamma(\lambda^*)>0$, *the steady state is locally stable.*
2. *If* $\Gamma(\lambda^*)<0$, *the steady state is locally unstable.*

From the aforementioned discussion, we can depict the dynamics of λ_t in Figure 1. The left-hand side of Figure 1 corresponds to the case when the steady state is stable, which shows that the equilibrium path is monotonic or fluctuating. The right-hand side of Figure 1 corresponds to the case where the steady state is unstable, which shows that periodical cycles can emerge. We consider the mechanism of these cycles in Subsection 3.4.

3.3. Effects of policy change

We now examine the effects of changes in policy variables on the steady-state growth and the local stability. From (20), (21), and (22), the steady-state

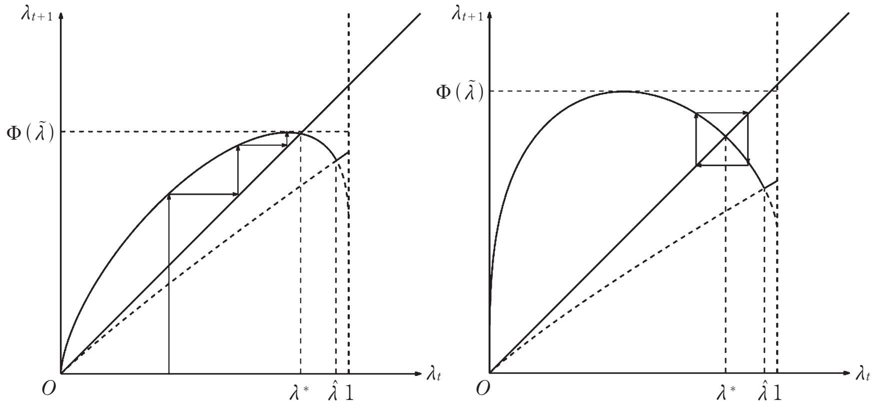


Figure 1: The dynamics of λ_t.

growth rate is as follows :

$$g^{*}=\frac{\rho}{\rho+(1-s_A)\theta}\left\{a_A(1-L_B)\left(\frac{1-\lambda_t}{\lambda_t}\right)^{1-\alpha}-[1-(1-s_A)\theta]\right\}=a_B L_B(\lambda^{*})^{\beta} \qquad (26)$$

Taking the total differentials of (25) yields

$$\frac{d\lambda^{*}}{dL_B}=-\frac{[\rho+(1-s_A)\theta]a_B(\lambda^{*})^{\beta}+\rho a_A\left(\frac{1-\lambda^{*}}{\lambda^{*}}\right)^{1-\alpha}}{\beta[\rho+(1-s_A)\theta]a_B L_B(\lambda^{*})^{\beta-1}+(1-\alpha)\rho a_A(1-L_B)\frac{(1-\lambda^{*})^{-\alpha}}{(\lambda^{*})^{2-\alpha}}}<0. \qquad (27)$$

By using (27), we differentiate (26) with respect to L_B as follows :

$$\frac{dg^{*}}{dL_B}=\frac{\rho a_A a_B(\lambda^{*})^{\alpha+\beta-2}(1-\lambda^{*})^{-\alpha}[-\beta L_B(1-\lambda^{*})+(1-\alpha)(1-L_B)]}{\beta[\rho+(1-s_A)\theta]a_B L_B(\lambda^{*})^{\beta-1}+(1-\alpha)\rho a_A(1-L_B)\frac{(1-\lambda^{*})^{-\alpha}}{(\lambda^{*})^{2-\alpha}}}.$$

The sign of $\frac{dg^{*}}{dL_B}$ is determined by the entity enclosed in the curly brackets of the numerator. Lemma 1's assumption that $0<L_B\leq 0.5$ and $\alpha+\beta\leq 1$ yields $L_B\leq 1-L_B$ and $\beta\leq 1-\alpha$, and thus, $\beta L_B(1-\lambda^{*})<(1-\alpha)(1-L_B)$ holds. Therefore, we obtain $\frac{dg^{*}}{dL_B}>0$. That is, an increase in L_B raises the steady-state growth rate.[3)]

Similarly, we investigate the effects of changes in s_A on the steady-state growth. Taking the total differentials of (25) yields

3) In this study, an increase in L_B crowds out the labor input into applied research. The result of $\frac{dg^{*}}{dL_B}>0$ is derived from the restriction of $L_B\in(0,\ 0.5)$. However, if the government increases L_B further, $\frac{dg^{*}}{dL_B}<0$ holds. That is, the relationship between the steady-state growth rate and L_B follows an inverted U-shape, and the steady-state growth-maximizing level of L_B exists. This result is similar to that of Park (1998), Gersbach et al. (2018), and Konishi (2018).

$$\frac{d\lambda^*}{ds_A}=\frac{\theta[\rho+1+a_BL_B(\lambda^*)^\beta]}{\beta[\rho+(1-s_A)\theta]a_BL_B(\lambda^*)^{\beta-1}+(1-\alpha)\rho a_A(1-L_B)\dfrac{(1-\lambda^*)^{-\alpha}}{(\lambda^*)^{2-\alpha}}}>0. \qquad (28)$$

By using (28), we differentiate (26) with respect to s_A as follows :

$$\frac{dg^*}{ds_A}=\beta a_BL_B(\lambda^*)^{\beta-1}\frac{d\lambda^*}{ds_A}>0.$$

Hence, a rise in s_A increases the steady-state growth rate.

Next, we examine the effects of changes in L_B on the local stability. From Proposition 1, differentiating $\Gamma(\lambda^*)$ with respect to L_B yields

$$\frac{d\Gamma\lambda^*}{dL_B}=\frac{2\rho+(1-s_A)\theta(1-\rho)}{(1-L_B)^2}-\rho a_A\frac{\alpha(1-\alpha)(\lambda^*)^{\alpha-2}}{(1-\lambda^*)^{\alpha+1}}\frac{d\lambda^*}{dL_B}$$
$$+\frac{(1-\beta)[\rho+(1-s_A)\theta]a_B(\lambda^*)^{\beta-1}}{(1-L_B)^2}\left[\lambda^*+\beta L_B(1-L_B)\frac{d\lambda^*}{dL_B}\right].$$

In the above equation, the first and second terms are positive. The sign of the third term is determined by the entity enclosed in square brackets. By using (27), we obtain

$$\lambda^*+\beta L_B(1-L_B)\frac{d\lambda^*}{dL_B}$$
$$=\frac{\beta[\rho+(1-s_A)\theta]a_BL_B^2(\lambda^*)^\beta+\rho a_A(1-L_B)\dfrac{(1-\lambda^*)^{-\alpha}}{(\lambda^*)^{\alpha-1}}[(1-\alpha)-\beta L_B(1-\lambda^*)]}{\beta[\rho+(1-s_A)\theta]a_BL_B(\lambda^*)^{\beta-1}+(1-\alpha)\rho a_A(1-L_B)\dfrac{(1-\lambda^*)^{-\alpha}}{(\lambda^*)^{2-\alpha}}}.$$

Analogous to the result of the steady-state growth rate, $\alpha+\beta\leq 1$ implies that $\beta L_B(1-\lambda^*)>0$ holds. Therefore, we obtain $\dfrac{d\Gamma(\lambda^*)}{dL_B}>0$. From this result and Proposition 1, an increase in L_B stabilizes the balanced growth path.

Similarly, we investigate the effects of changes in s_A on the local stability. We differentiate $\Gamma(\lambda^*)$ with respect to s_A as follows :

$$\frac{d\Gamma\lambda^*}{ds_A} = -\frac{\theta(1-\rho)}{1-L_B} - \rho a_A \frac{\alpha(1-\alpha)(\lambda^*)^{\alpha-2}}{(1-\lambda^*)^{\alpha+1}} \frac{d\lambda^*}{ds_A}$$
$$-\frac{(1-\beta)a_B L_B(\lambda^*)^{\beta-1}}{1-L_B}\left\{\theta\lambda^* + \beta[\rho+(1-s_A)\theta]\frac{d\lambda^*}{ds_A}\right\}.$$

The first and second terms are negative. The sign of the third term is determined by the entity enclosed in square brackets. By using (28), we obtain

$$\theta\lambda^* + \beta[\rho+(1-s_A)\theta]\frac{d\lambda^*}{ds_A}$$
$$= \theta \frac{(1-\alpha)\rho a_A(1-L_B)\dfrac{(1-\lambda^*)^{-\alpha}}{(\lambda^*)^{\alpha-1}} - \beta[\rho+(1-s_A)\theta](\rho+1)}{\beta[\rho+(1-s_A)\theta]a_B L_B(\lambda^*)^{\beta-1} + (1-\alpha)\rho a_A(1-L_B)\dfrac{(1-\lambda^*)^{-\alpha}}{(\lambda^*)^{2-\alpha}}}. \tag{29}$$

As shown in Appendix C, the condition that the numerator is positive is expressed as follows :

$$a_A > \frac{2\beta(\rho+\theta)(\rho+1)}{(1-\alpha)\rho} a_A(1-\alpha)^{1-\alpha} \tag{30}$$

Hence, a sufficiently high a_A implies that $\dfrac{d\Gamma(\lambda^*)}{ds_A} < 0$ holds. From this result and Proposition 1, a rise in s_A destabilizes the balanced growth path. In summary, we can state the following proposition :

Proposition 2. Suppose that $0 < L_B \leq 0.5$, $\alpha+\beta \leq 1$, and $a_A > \dfrac{2\beta(\rho+\theta)(\rho+1)}{(1-\alpha)\rho} a_A(1-\alpha)^{1-\alpha}$.

1. An increase in L_B increases both the steady-state growth rate and the possibility of cyclical instability.
2. An increase in s_A increases the steady-state growth rate and reduces the possibility of cyclical instability.

Proposition 2 implies that basic research spending and subsidy for applied research raise the steady-state growth rate under certain conditions. However, these two policies have different effects on the local stability. That is, an increase in basic research investment stabilizes the economy, while an increase in subsidy for applied research destabilize the economy. We discuss the mechanism of cycles in the following subsection.

3.4. Mechanism of cycles

Next, we consider the mechanism of cycles. By using (19) and (20), when λ_t is relatively higher, both the wage rate for skilled labor W_t^s and the amount of skilled labor devoted to applied research $L_{A,t}$ are lower. The reasoning is as follows. From the definition of λ_t and W_t, as λ_t increases, the pool of ideas awaiting commercialization W_t decreases, and thus, the knowledge spillover in the applied research sector also decreases. Because the productivity of applied research decreases, the demand for skilled labor in the applied research sector reduces, and consequently, the wage rate for skilled labor also decreases. From (5) and (8), a reduction in the wage rate for skilled labor decreases good prices, thus increasing their demand and reallocating skilled labor from applied research to the production of goods. Therefore, a higher λ_t implies that the growth in the number of differentiated goods g_t^A is lower, as shown in (21). In the basic research sector, on the other hand, the knowledge spillover from applied research is larger when λ_t is relatively higher. Hence, there is increased growth in the number of ideas that have been generated through basic research g_t^B, as shown in (22). From these results, a lower g_t^A and a higher g_t^B reduce λ_{t+1}. When the reduction in λ_{t+1} is sufficiently large, the low value of λ_{t+1} in turn accelerates applied research and hinders basic research. Therefore, g_{t+1}^A is higher and g_{t+1}^B is lower, thus increasing λ_{t+2}. This process occurs repeatedly along the equilibrium path, and thus, two-period cycles emerge.

4. Numerical example

To demonstrate the equilibrium path more clearly, we employ a numerical example. First, we show the effect of L_B. We choose the following parameters: $\alpha = 0.6$, $\beta = 0.4$, $a_A = 4.5$, $a_B = 1.5$, $\rho = 0.25$, $\theta = 0.3$, and $s_A = 0.2$. Figure 2 represents the bifurcation diagram for L_B and shows the emergence of two-period cycles depending on the values of L_B. The vertical axis shows the value of $\lambda_t\,(0 < \lambda_t < 1)$ and the horizontal axis shows the value of $L_B\,(0.1 < L_B < 0.3)$. When L_B is sufficiently large, a unique limit point exists. When L_B decreases, two -period cycles and endogenous fluctuations are observed. In addition, Figures 3 (a) and (b) depict the dynamics of λ_t. Figure 3 (a) corresponds to the case in which $L_B = 0.2$; this shows that the steady state is unstable and that two-period cycles emerge. Figure 3 (b) corresponds to the case in which $L_B = 0.21$; this shows that the steady state is stable and that the equilibrium path fluctuates.

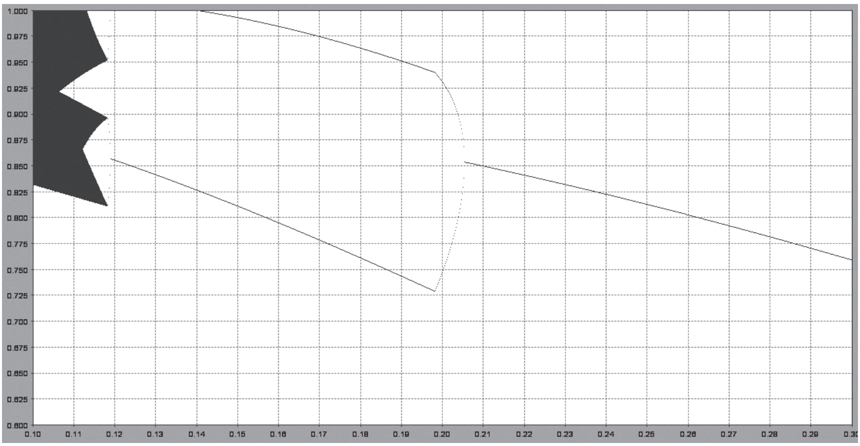
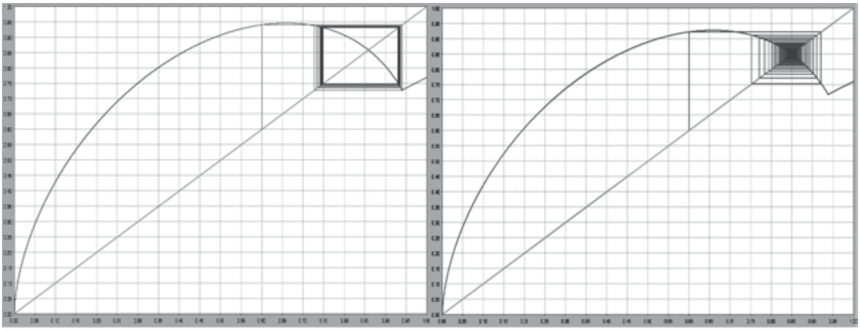


Figure 2: The bifurcation diagram for L_B



(a) $L_B = 0.2$　　　　(b) $L_B = 0.21$

Figure 3 : The dynamics of λ_t for each value of L_B

Next, we show the effect of s_A. We choose the following parameters : $\alpha = 0.6$, $\beta = 0.4$, $a_A = 4.5$, $a_B = 1.5$, $\rho = 0.25$, $\theta = 0.3$, and $L_B = 0.2$. Figure 4 represents the bifurcation diagram for s_A and shows the emergence of two-period cycles depending on the values of s_A. The vertical axis shows the value of $\lambda_t\,(0 < \lambda_t < 1)$ and the horizontal axis shows the value of $s_A\,(0 < s_A < 0.4)$.

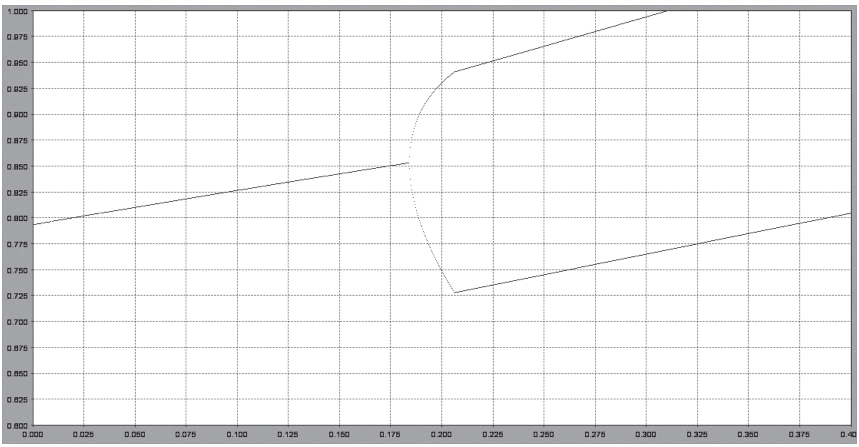
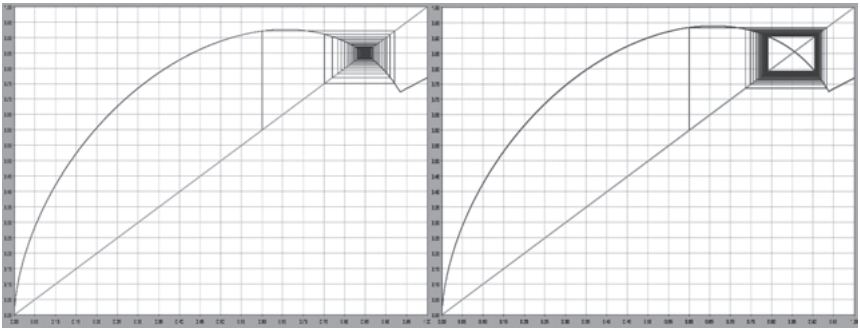


Figure 4 : The bifurcation diagram for s_A



(a) $s_A = 0.17$ (b) $s_A = 0.19$

Figure 5 : The dynamics of λ_t for each value of s_A

When s_A is sufficiently small, a unique limit point exists. When s_A increases, two-period cycles and endogenous fluctuations are observed. In addition, Figures 5 (a) and (b) depict the dynamics of λ_t. Figure 5 (a) corresponds to the case in which $s_A = 0.17$; this shows that the steady state is stable and that the equilibrium path fluctuates. Figure 5 (b) corresponds to the case in which $s_A = 0.19$; this shows that the steady state is unstable and that two-period cycles emerge.

5. Conclusion

In this study, we developed a variety-expansion growth model that integrated the applied and basic research sectors to examine growth cycles. We show that the equilibrium path can exhibit two-period cycles through the interplay between applied and basic research. In addition, we explore the effects of change in basic research spending and applied research subsidy. The steady-state growth rate increases when basic research spending or applied research subsidy increases. However, the effects on the possibility of cyclical instability differ by these two policies. An increase in basic research spending reduces the possibility of cyclical instability, while an increase in applied research subsidy raises the possibility of cyclical instability.

Acknowledgments

I thank the participants at the 16th Workshop in Macroeconomics for Young Economists and the 2015 Autumn Annual Meeting of the Japanese Economic Association in Sophia University for their useful comments. This work was supported by the Research Fellowship for Young Scientists of the Japan Society for the Promotion of Science (JSPS) (No. 16J09472) and the 2019 Special Research Grant from Matsuyama University. Any errors are my responsibility.

Appendix

A. Proof of Lemma 1

Differentiating $\Phi(\lambda)$ with respect to λ yields

$$\Phi'(\lambda)=\frac{\Lambda(\lambda)}{[\rho+(1-s_A)\theta](1+a_B L_B \lambda^{\beta})^2}, \tag{A. 1}$$

Where

$$\Lambda(\lambda)\equiv \rho a_A(1-L_B)\frac{\alpha-\lambda}{\lambda^{\alpha}(1-\lambda)^{\alpha}}+\rho[1-(1-s_A)\theta][1+(1-\beta)a_B L_B \lambda^{\beta}]$$
$$+\rho a_A a_B L_B(1-L_B)\frac{\lambda^{\alpha+\beta-1}}{(1-\lambda)^{\alpha}}[\alpha-\beta-(1-\beta)\lambda].$$

We differentiate $\Lambda(\lambda)$ with respect to λ as follows :

$$\Lambda'(\lambda)=\rho\lambda^{\alpha}\left\{\frac{\alpha(1-\alpha)a_A(1-L_B)}{(1-\lambda)^{\alpha+1}}+\beta(1-\beta)[1-(1-s_A)\theta]a_B L_B \lambda^{1-\alpha+\beta}\right\}$$
$$+\rho a_A a_B L_B(1-L_B)\frac{\lambda^{\alpha+\beta-1}}{(1-\lambda)^{\alpha}}[(\alpha-\beta(\alpha+\beta-1)-\beta(1-\beta)\lambda].$$

The second term's entity within curly brackets is negative if $\alpha \geq \beta$ and $\alpha+\beta \leq 1$. Then, the condition that the first term's square bracket is as follows :

$$\frac{a_A}{a_B} > \frac{\beta(1-\beta)}{\alpha(1-\alpha)}[1-(1-s_A)\theta]\frac{L_B}{1-L_B}\lambda^{1-\alpha+\beta}(1-\lambda)^{\alpha+1}. \tag{A. 2}$$

Let us define $\psi(\lambda) \equiv \lambda^{1-\alpha+\beta}(1-\lambda)^{\alpha+1}$. Differentiating $\psi(\lambda)$ with respect to λ yields

$$\psi'(\lambda) = \lambda^{-\alpha+\beta}(1-\lambda)^{\alpha}[1-\alpha+\beta-(2+\beta)\lambda].$$

$\lambda \lesseqgtr \frac{1-\alpha+\beta}{2+\beta}$ implies that $\psi'(\lambda) \gtreqless 0$ holds, and thus, $\lambda = \frac{1-\alpha+\beta}{2+\beta}$ maximizes $\psi(\lambda)$. From $L_B \in (0,\ 0.5]$, $s_A \in [0,\ 1)$, and this result, the sufficient condition of (A. 2) is as follows:

$$\frac{a_A}{a_B} > \frac{\beta(1-\beta)}{\alpha(1-\alpha)}\left(\frac{1-\alpha+\beta}{2+\beta}\right)^{1-\alpha+\beta}\left(\frac{1+\alpha}{2+\beta}\right)^{\alpha+1} \equiv \Delta(\alpha,\ \beta). \tag{A. 3}$$

Figure A. 1 illustrates the combinations of α and β that maximize $\Delta(\alpha,\ \beta)$. By using this result, we can depict the relation between α and the maximum value of $\Delta(\alpha,\ \beta)$ as shown in Figure A. 2. From Figure A. 2, $\Delta(\alpha,\ \beta)$ is lower than one if $\alpha \in [0.05,\ 0.9]$. That is, the sufficient condition of (A. 3) is that $0.05 \leq \alpha \leq 0.9$ and $a_A \geq a_B$. In summary, $\Lambda'(\lambda) < 0$ holds if we assume that $0 < L_B \leq 0.5$, $0.05 \leq \alpha \leq 0.9$, $\alpha \geq \beta$, $\alpha + \beta \leq 1$, and $a_A \geq a_B$. Moreover, we investigate the value of $\Lambda(\lambda)$ at $\lambda \to 0$ and $\lambda \to 1$ as follows:

$$\lim_{\lambda \to 0} \Lambda(\lambda) = +\infty \quad \textit{and} \quad \lim_{\lambda \to 1} \Lambda(\lambda) = -\infty \tag{A. 4}$$

Hence, there is $\tilde{\lambda}$ which satisfies $\Lambda(\tilde{\lambda}) = 0$ and $\tilde{\lambda} \in (0,\ 1)$. Because $\lambda \lesseqgtr \tilde{\lambda}$ implies $\Lambda(\tilde{\lambda}) \gtreqless 0$, we obtain $\Phi'(\lambda) \gtreqless 0$ when $\lambda \lesseqgtr \tilde{\lambda}$. In addition, from (A. 1) and (A. 4), $\lim_{\lambda \to 0} \Phi'(\lambda) = +\infty$ holds, and thus, the steady-state $\lambda^* = 0$ is surely unstable.

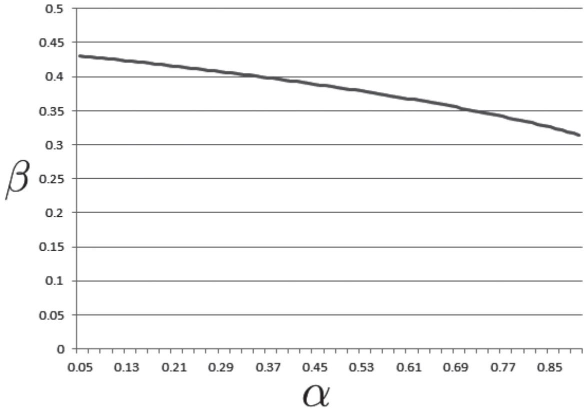


Figure A. 1 : The combinations of α and β that maximize $\Delta(\alpha, \beta)$

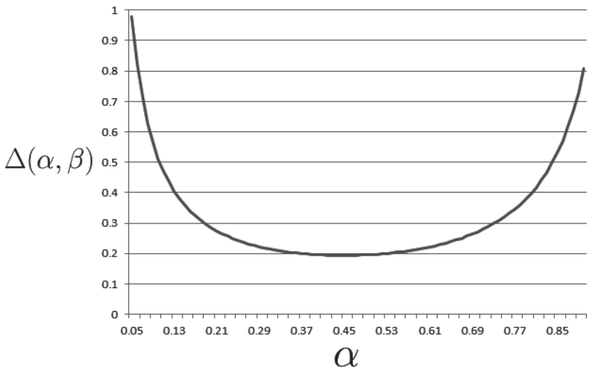


Figure A. 2 : The relation between α and the maximum value of $\Delta(\alpha, \beta)$

We then consider the dynamic system when the applied research is not conducted. From (24), differentiating $\Psi(\lambda)$ with respect to λ yields

$$\Psi'(\lambda) = \frac{1+(1-\beta)a_B L_B \lambda^{\beta}}{(1+a_B L_B \lambda^{\beta})^2}.$$

Thus, $0 < \Psi'(\lambda) < 1$ holds. By using (24), the properties of $\Phi(\lambda)$ and $\Psi(\lambda)$ we obtain

$$\Phi(0) = 0 \quad and \quad \Phi(1) = \frac{\rho(1-\theta)}{(\rho+\theta)(1+a_B L_B)}, \tag{A. 5}$$

$$\Psi(0) = 0 \quad and \quad \Psi(1) = \frac{1}{1+a_B L_B}. \tag{A. 6}$$

(A. 5) and (A. 6) imply that $\Phi(1) < \Psi(1)$ holds. Therefore, we confirm that the unique intersection of $\Phi(\lambda)$ and $\Psi(\lambda)$ is in the region where $0 < \lambda < 1$.

B. Proof of Proposition 1

By using (A. 1) and (25), we obtain

$$\Phi'(\lambda^*) = 1 - \frac{(1-\alpha)a_A(1-L_B)\left(\frac{\lambda^*}{1-\lambda^*}\right)^{\alpha}}{a_A(1-L_B)(\lambda^*)^{\alpha}(1-\lambda^*)^{1-\alpha} + [1-(1-s_A)\theta]\lambda^*} - \frac{\beta a_B L_B(\lambda^*)^{\beta}}{1+a_B L_B(\lambda^*)^{\beta}}.$$

Therefore, $\Phi'(\lambda^*) < 1$ holds. If $\Phi'(\lambda^*) < -1$, the steady state is locally unstable. On the other hand, if $\Phi'(\lambda^*) > -1$, the steady state is locally stable. From (A. 1) and (25), we rearrange $\Phi'(\lambda^*) < -1$ as follows:

$$\Gamma(\lambda^*) \equiv \frac{2\rho + (1-s_A)\theta(1-\rho)}{1-L_B} + \frac{\rho a_A(\alpha-\lambda^*)}{(\lambda^*)^{1-\alpha}(1-\lambda^*)^{\alpha}} + \frac{(1-\beta)[\rho+(1-s_A)\theta]a_B L_B(\lambda^*)^{\beta}}{1-L_B} < 0.$$

In contrast, $\Phi'(\lambda^*) > -1$ holds if $\Gamma(\lambda^*) > 0$.

C. Derivation of the condition of (30)

By using (29), the condition of $\theta\lambda^* + \beta[\rho+(1-s_A)\theta]\frac{d\lambda^*}{ds_A} > 0$ is as follows:

$$a_A > \frac{\beta[\rho+(1-s_A)\theta](\rho+1)}{(1-\alpha)\rho(1-L_B)}(\lambda^*)^{1-\alpha}(1-\lambda^*)^{\alpha} \tag{C. 1}$$

Let us define $\phi(\lambda)\equiv\lambda^{1-\alpha}(1-\lambda)^{\alpha}$. Differentiating $\phi(\lambda)$ with respect to λ yields

$$\phi'(\lambda)=\lambda^{-\alpha}(1-\lambda)^{\alpha-1}(1-\alpha-\lambda).$$

$\lambda\lesseqqgtr 1-\alpha$ implies that $\phi'(\lambda)\gtreqqless 0$ holds, and thus, $\lambda=1-\alpha$ maximizes $\phi(\lambda)$. From $L_B\in(0,\ 0.5]$, $s_A\in[0,\ 1)$, and this result, the sufficient condition of (C. 1) is as follows :

$$a_A>\frac{2\beta(\rho+\theta)(\rho+1)}{(1-\alpha)\rho}\alpha^{\alpha}(1-\alpha)^{1-\alpha}.$$

References

[1] Aloi, M. and L. Lasselle (2007) Growth and welfare effects of stabilizing innovation cycles. *Journal of Macroeconomics* 29, 806-823.

[2] Chu, A., G. Cozzi, and S. Galli (2012) Does intellectual monopoly stimulate or stifle innovation? *European Economic Review* 56, 727-746.

[3] Chu, A. and Y. Furukawa (2013) Patentability and knowledge spillovers of basic R&D. *Southern Economic Journal* 79, 928-945.

[4] Cohen, W. M., R. R. Nelson, and J. P. Walsh (2002) Links and impacts : the influence of public research on industrial R&D. *Management Science* 48, 1-23.

[5] Cozzi, G. and S. Galli (2009) Science-based R&D in Schumpeterian growth. *Scottish Journal of Political Economy* 56, 474-491.

[6] Cozzi, G. and S. Galli (2013) Privatization of knowledge : Did the U. S. get it right? *Economics Working Paper Series* 1307, University of St. Gallen, School of Economics and Political Science.

[7] Cozzi, G. and S. Galli (2014) Sequential R&D and blocking patents in the dynamics of growth. *Journal of Economic Growth* 19, 183-219.

[8] Deissenberg, C. and J. Nyssen (1998) A simple model of Schumpeterian growth with complex dynamics. *Journal of Economic Dynamics and Control* 22, 247-266.

[9] Diamond, P. A. (1965) National debt in a neoclassical growth model. *American Economic Review* 55, 1126-1150.

[10] Francois, P. and H. Lloyd-Ellis (2003) Animal spirits through creative destruction. *American Economic Review* 93, 530-550.

[11] Francois, P. and H. Lloyd-Ellis (2008) Implementation cycles, investment, and growth. *International Economic Review* 49, 901-942.

[12] Francois, P. and H. Lloyd-Ellis (2009) Schumpeterian cycles with pro-cyclical R&D. *Review of Economic Dynamics* 12, 567-591.

[13] Francois, P. and S. Shi (1999) Innovation, growth, and welfare-improving cycles. *Journal of Economic Theory* 85, 226-257.

[14] Freeman, S., D. P. Hong, and D. Peled (1999) Endogenous cycles and growth with indivisible technological developments. *Review of Economic Dynamics* 2, 403-432.

[15] Gale, D. (1996) Delay and cycles. *Review of Economic Studies* 63, 169-198.

[16] Gersbach, H. and M. Schneider (2015) On the global supply of basic research. *Journal of Monetary Economics* 75, 123-137.

[17] Gersbach, H., M. Schneider, and O. Schneller (2013) Basic research, openness, and convergence. *Journal of Economic Growth* 18, 33-68.

[18] Gersbach, H., G. Sorger, and C. Amon (2018) Hierarchical growth: Basic and applied research. *Journal of Economic Dynamics and Control* 90, 434-459.

[19] Grandmont, J. M. (2008) Nonlinear difference equations, bifurcations and chaos: An introduction. *Research in Economics* 62, 122-177.

[20] Griliches, Z. (1986) Productivity, R&D, and basic research at the firm level in the 1970s. *American Economic Review* 76, 141-154.

[21] Grossman, G. M. and E. Helpman (1991) *Innovation and growth in the global economy*. MIT Press, Cambridge, MA.

[22] Growiec, J. and I. Schumacher (2013) Technological opportunity, long-run growth, and convergence. *Oxford Economic Papers* 65, 323-351.

[23] Haruyama, T. (2009) R&D policy in a volatile economy. *Journal of Economic Dynamics and Control* 33, 1761-1778.

[24] Jaffe, A. B. (1989) Real effects of academic research. *American Economic Review* 79, 957-970.

[25] Konishi, K. (2018) Basic and applied research: A welfare analysis. *Japanese Economic Review* 69, 414-437.

[26] Li, B. and J. Zhang (2014) Subsidies in an economy with endogenous cycles over investment and innovation regimes. *Macroeconomic Dynamics* 18, 1351-1382.

[27] Li, C. W. (2001) Science, diminishing returns and long waves. *Manchester School* 69, 553-573.

[28] Mansfield, E. (1998) Academic research and industrial innovation: An update of empirical findings. *Research Policy* 26, 773-776.

[29] Matsuyama, K. (1999) Growing through cycles. *Econometrica* 67, 335-347.

[30] Park, W. G. (1998) A theoretical model of government research and growth. *Journal of Economic Behavior and Organization* 34, 69-85.

［31］ Shleifer, A.（1986） Implementation cycles. *Journal of Political Economy* 94, 1163-1190.

［32］ Wälde, K.（2005） Endogenous growth cycles. *International Economic Review* 46, 867-894.