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Economic Growth and Unrealized Capital Gains Taxation on Land

Katsuhiro Aono

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1. Introduction

In the earlier paper (Aono (2015)), we integrated a land asset market into the economic growth model and examined the characteristics of the steady state growth path. In the previous paper (Aono (2016)), we examined the stability of the long-run steady state path, and investigate the paths which do not converge to the long-run steady state in greater detail. In this paper, we ask what happens when we impose unrealized capital gains tax on land.

The rest of the paper is organized as follows. Section 2 outlines the basic structure of the model, and derives the dynamic equations of the economy. Section 3 examines the characteristics of the equilibrium growth paths in the case where unrealized capital gains tax on land is imposed. Section 4 summarizes the main results in this paper, and discusses possible extensions.

2. Model

2.1. The basic model

The basic model and notation in this paper are as follows.

* Former Professor/Matsuyama University, Dr. Economics (Kobe University)

1) This paper is a revised version of Aono (1976). In this paper we mainly focus on the effects of capital gains on land from the view point of equity and efficiency.

(a) Technology.

Output, Y is a function of inputs of labor, N , capital, K , and land, L . The production function for this output is twice differential, homogeneous of degree one, in addition, it has the following properties : (i) positive and diminishing marginal productivities of the three factors, (ii) impossibility of any product without one of the factors. Technical change is assumed to be land-augmenting and labor-augmenting. The rate of land-augmenting technical progress and the rate of labor-augmenting technical progress are denoted as h and α , respectively. The above assumptions are summarized by

$$Y = F(K, N^*, L^*), F_K, F_{N^*}, F_{L^*} > 0, F_{KK}, F_{N^*N^*}, F_{L^*L^*} < 0, \quad (1)$$

$$L^* = L e^{ht}, \quad (2)$$

$$N^* = N e^{\alpha t}. \quad (3)$$

(b) The supply of land and labor.

Land is assumed to be fixed in supply, and labor is assumed to grow at the given exponential rate, n . Thus the sum of the rate of labor-augmenting technical progress and the rate of growth of labor is $\alpha + n$. We assume that the following conditions are satisfied.

$$L = \bar{L}, \bar{L} : \text{constant}, \quad (4)$$

$$N = N_0 e^{nt}, \quad (5)$$

$$0 < \alpha + n < 1. \quad (6)$$

(c) Land asset market

Consider first a new portfolio equation when an unrealized capital gains tax is imposed on land. Denoting the rate of the unrealized capital gains tax as τ , landowners expect $(1 - \tau)\dot{P}^e$ capital gains on land in each period when they hold land. For analytical simplicity, we assume that there exists a perfect land asset

market. Portfolio equilibrium requires that alternative investment options yield the same net rate of return. Since capital is the only asset other than land, and since these two assets have the same risk properties under the assumption of a perfect land asset market, we obtain the following equilibrium condition ;

$$(1 - \tau) \frac{\dot{P}^e}{P} + \frac{\rho^e}{P} = r^e, \quad (7)$$

where P is the price of land in terms of goods, \dot{P}^e is the expected change of land prices, ρ^e is the expected rent of land, and r^e is the expected rate of return on capital. The equation (7) tells us that the land asset market is in equilibrium when the expected rate of yield in land prices, $(1 - \tau)\dot{P}^e/P$ plus the expected rent-price ratio, ρ^e/P are equal to the expected rate of return on capital, r^e .

For simplicity, we assume that expected and actual price changes are identical, i. e.

$$\frac{\dot{P}^e}{P} = \frac{\dot{P}}{P}, \quad \rho^e = \rho, \quad r^e = r. \quad (8)$$

It is important to consider the implications and the limitations of the equation (7) and (8). We do not assume that landowners have a long-run perfect foresight because they do not know what will happen in the long and distant future. We assume that landowners adjust their expectations instantaneously for analytical simplicity.

(d) Consumption function and savings function

We now consider a new consumption function, and hence the savings function. We shall assume that landowners consume or save out of capital gains minus the capital gains tax on land $(1 - \tau)\dot{P}L$ and that the revenues out of the capital gains tax on land are spent on consumption or investment. We also assume a generalized Cambridge savings function instead of a proportional savings function. For

analytical simplicity, we assume that workers do not own their land, and that they do not have to pay the rental rate of land to landowners. Denote the rate of government savings as s_G . Then, we get a new consumption function ;

$$C = Y - \{s_\pi(r + \delta)K + s_W RN + s_{L1}\rho L\} + (1 - s_{L2})(1 - \tau)\dot{P}L - s_G \tau \dot{P}L$$

with $0 < s_W \leq s_{L1} = s_\pi \leq s_{L2} < 1$, $0 \leq s_G \leq 1$. (9)

where s_π : the saving rate of capitalists (entrepreneurs) out of profits, s_W : the saving rate of workers, s_{L1} : the saving rate of landowners out of rents, s_{L2} : the saving rate of landowners out of capital gains minus the capital gains tax on land. δ : the rate of depreciated capital. r : the net rate of return on capital, R : the real wage rate, ρ : the rental rate of land.

Since total output, Y is equal to consumption, C plus capital depreciation, δK , plus net investment, \dot{K} , we get

$$Y = C + \delta K + \dot{K}. \quad (10)$$

Substituting (9) into (10) yields

$$\dot{K} + \delta K = \{s_\pi(r + \delta)K + s_W RN + s_{L1}\rho L\} - (1 - s_{L2})(1 - \tau)\dot{P}L + s_G \tau \dot{P}L$$

with $0 \leq s_W < s_{L1}$, $s_\pi \leq s_{L2} < 1$, $0 \leq s_G \leq 1$. (11)

(e) Determination of factor prices.

Given R , ρ and the production function which is homogeneous of degree one (exhibits constant returns to scale), a competitive producer can maximize the net rate of return on capital when the marginal productivities of capital, F_K , labor, F_N and land, F_L are equal to $r + \delta$, R , and ρ respectively. Thus, we get

$$F_K = r + \delta, \quad (12)$$

$$F_N = R, \quad (13)$$

$$F_L = \rho. \quad (14)$$

2.2. Derivation of dynamic equations

It turns out that the dynamics of economy can best be described in terms of the ratios, $l \equiv PL/N^*$, $u \equiv L^*/N^*$ and $k \equiv K/N^*$.

From the equation (1), we obtain

$$\frac{Y}{N^*} = F(K/N^*, 1, L^*/N^*).$$

Thus, letting $y = Y/N^*$, the production function can be rewritten as a function of k and u .

$$y = f(k, u). \quad (15)$$

Differentiating the above equation with respect to K , N^* and L^* , we get

$$F_K = f_k(k, u), \quad (16)$$

$$F_{N^*} = f(k, u) - kf_k(k, u) - uf_u(k, u), \quad (17)$$

$$F_{L^*} = f_u(k, u). \quad (18)$$

For analytical simplicity, we shall assume that u is constant. Since $u = L^*/N^*$, the assumption implies that technical change is of such a character that the rate of land-augmenting technical progress is equal to the sum of the rate of growth of labor supply and the rate of labor-augmenting technical progress. Thus, F_K , F_{N^*} and F_{L^*} can be written as a function of k .

Differentiating $l \equiv PL/N^*$ with respect to time t , we get

$$\hat{l} = \hat{P} + \hat{L} - \hat{N}^*, \quad (19)$$

where $\hat{l} = \dot{l}/l$, $\dot{l} = d\dot{l}/dt$. Substituting (7) and (8) into (19), and using (3), (4), (5), (12), (14), (15), (16) and (18), we obtain

$$\begin{aligned} \hat{l} &= \frac{1}{(1-\tau)} \left(-\frac{uf_u}{l} + f_k - \delta \right) - (\alpha + n). \\ \dot{l} &= -\frac{uf_u}{(1-\tau)} + \left\{ \frac{f_k - \delta}{(1-\tau)} - (\alpha + n) \right\} l. \end{aligned} \quad (20)$$

We shall now derive $\dot{k}(k, l)$. From the definition of $k \equiv K/N^*$, we get

$$\dot{k} = \dot{K} - \dot{N}^* = \dot{K} - (\alpha + n). \quad (21)$$

From the equation (11), we obtain

$$\dot{K} + \delta = \left\{ s_\pi(r + \delta) + s_W R \left(\frac{N}{K} - \rho \frac{L}{K} \right) + s_{L1} \rho \frac{L}{K} \right\} - (1 - s_{L2})(1 - \tau) \frac{\dot{P}L}{K} + s_G \tau \frac{\dot{P}L}{K}.$$

Using (4), (12), (13), (14), $F_L = F_{L^*} e^{h1}$ and $F_N = F_{N^*} e^{at}$, the above equation can be rewritten as

$$\begin{aligned} \dot{K} + \delta &= s_\pi F_K + s_W \left(\frac{F_{N^*}}{k} - F_{L^*} \frac{u}{k} \right) + s_{L1} F_{L^*} \frac{u}{k} - (1 - s_{L2})(1 - \tau) \dot{P} \frac{l}{k} \\ &\quad + s_G \tau \dot{P} \frac{l}{k}. \end{aligned} \quad (22)$$

Also, \dot{P} can be rewritten as

$$\dot{P} = \frac{1}{(1 - \tau)} \left(-\frac{u F_{L^*}}{l} + F_K - \delta \right) \quad (23)$$

Substituting (23) into (22) and using (16), (17) and (18), we get

$$\begin{aligned} \dot{K} + \delta &= s_\pi f_k + s_W (f - k f_k - u f_u) \frac{1}{k} + s_{L1} f_u \frac{u}{k} - (1 - s_{L2})(1 - \tau) \left(-\frac{u f_u}{l} + f_k - \delta \right) \frac{l}{k} \\ &\quad + s_G \tau \left(-\frac{u f_u}{l} + f_k - \delta \right) \frac{l}{k}. \end{aligned} \quad (24)$$

Substitute (24) into (21), we obtain

$$\begin{aligned} \dot{k} &= s_\pi k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u + (1 - s_{L2})(1 - \tau) u f_u \\ &\quad - (1 - s_{L2})(1 - \tau)(f_k - \delta)l - s_G \tau u f_u + s_G \tau (f_k - \delta)l - (\delta + \alpha + n)k. \end{aligned} \quad (25)$$

The dynamic behavior of the system is thus described by (20) and (25).

3. Characteristics of the equilibrium growth paths in the case where unrealized capital gains tax on land is imposed

3.1. The equilibrium growth path when there is no capital gains tax

In this section, we shall examine some qualitative properties of the equilibrium growth path in the case where unrealized capital gains tax on land is imposed. For this purpose, we first examine the equilibrium growth path in the case where capital gains tax is not imposed.

Letting $\tau = 0$ in (20), we get the $\dot{l} = 0$ curve :

$$l = \frac{uf_u}{f_k - (\delta + \alpha + n)}. \quad (26)'$$

It is easily verified that (26)' has the following properties.

$$l_k(k)|_{l=0} = \frac{[uf_{uk}\{f_k - (\delta + \alpha + n)\} - f_{kk}uf_u]}{\{f_k - (\delta + \alpha + n)\}^2} > 0, \quad (27)'$$

$$\lim_{k \rightarrow 0} l(k)|_{l=0} = 0, \quad (28)'$$

and

$$\lim_{k \rightarrow \tilde{k}} l(k)|_{l=0} = \infty. \quad (29)'$$

In the case where $k = \tilde{k}$, $f_k(\tilde{k}) = \delta + \alpha + n$. Therefore, in Figure 1, along the vertical line $f_k(\tilde{k}) = \delta + \alpha + n$, there is no change in k . Above the $\dot{l} = 0$ curve, l increases, below it, l decreases.

We now examine the $\dot{k} = 0$ curve and its properties. Letting $\tau = 0$ in (25), we get the $\dot{k} = 0$ curve :

$$l = \frac{s_\pi k f_k - (\delta + \alpha + n)k + s_W(f - k f_k - u f_u) + s_{L1} u f_u + (1 - s_{L2}) u f_u}{(1 - s_{L2})(f_k - \delta)}. \quad (30)'$$

Differentiating (30)' with respect to k , we derive the formula for the slope of the $\dot{k} = 0$ curve :

$$l_k(k)|_{\dot{k}=0} = \frac{(f_k - \delta)}{(1 - s_{L2})(f_k - \delta)^2} \{ s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W) k f_{kk} \\ + (s_{L1} - s_W) u f_{uk} + (1 - s_{L2}) u f_{uk} \} - (1 - s_{L2}) f_{kk} l. \quad (31)'$$

Since the denominator is positive, it follows that the sign of (31)' depends only upon the numerator. Although the numerator is unsigned, we see that $f_k \rightarrow \infty$ for $k \rightarrow 0$ and that the $\dot{k} = 0$ curve meets the k -axis (see (30)').

From (30)' we also see that the $\dot{k} = 0$ curve is at its maximum level at l^{**} , the height of the $\dot{k} = 0$ curve at k^{**} , where, as Figure 1 makes clear, l^{**} is defined by :

$$l^{**} = \frac{s_\pi k f_k - (\delta + \alpha + n) k + (s_\pi - s_W) k f_{kk} + (s_{L1} - s_W) u f_{uk}}{(1 - s_{L2}) f_{kk}}. \quad (32)'$$

Above the $\dot{k} = 0$ curve, k decreases, below it, k increases.

As Figure 1 shows, the $\dot{k} = 0$ curve is the inverted U shaped locus, the $\dot{l} = 0$ curve cuts the $\dot{k} = 0$ curve from below and there is a unique equilibrium. (see (Aono (2016b) **Figure 1. Phase diagram of the dynamic behavior of the system.**) We examined the stability of the long-run steady state path, and investigated the paths which do not converge to the long-run steady state in greater detail.)

3.2. The equilibrium growth path when capital gains tax is imposed

3.2.1. In the case where the revenues out of the capital gains tax on land are spent on consumption

We shall now examine the properties of the equilibrium growth path when capital gains tax is imposed. Firstly, we assume that the revenues out of the capital gains tax on land are entirely spent on consumption ($s_G = 0$). The dynamic behavior of the system can be illustrated in a phase diagram, as in Figure 1. In

order to examine the equilibrium growth path, we first derive the $\dot{l} = 0$ curve. From (20), we see that $\dot{l} = 0$ and $l \neq 0$ if and only if

$$l = \frac{uf_u}{(f_k - \delta) - (1 - \tau)(\alpha + n)}. \quad (26)$$

It is easily verified that (26) has the following properties.

$$l_k(k)|_{l=0} = \frac{[uf_{uk}\{(f_k - \delta) - (1 - \tau)(\alpha + n)\} - f_{kk}uf_u]}{\{(f_k - \delta) - (1 - \tau)(\alpha + n)\}^2} > 0. \quad (27)$$

$$\lim_{k \rightarrow 0} l(k)|_{l=0} = 0. \quad (28)$$

$$\lim_{k \rightarrow \tilde{k}} l(k)|_{l=0} = \infty. \quad (29)$$

In the case where $k = \tilde{k}$, $f_k(\tilde{k}) = \delta + (1 - \tau)(\alpha + n)$. Therefore, in Figure 1, along the vertical line $f_k(\tilde{k}) = \delta + (1 - \tau)(\alpha + n)$ there is no change in k . Above the $\dot{l} = 0$ curve, l increases, below it, l decreases.

We now examine the $\dot{k} = 0$ curve and its properties. From (25), we get the $\dot{k} = 0$ curve :

$$l = \frac{s_\pi k f_k - (\delta + \alpha + n)k + s_W(f - k f_k - u f_u) + s_{L1} u f_u + (1 - \tau)(1 - s_{L2}) u f_u}{(1 - \tau)(1 - s_{L2})(f_k - \delta)} \quad (30)$$

Differentiating (30) with respect to k , we derive the formula for the slope of the $\dot{k} = 0$ curve :

$$l_k(k)|_{\dot{k}=0} = \frac{(1 - \tau)(1 - s_{L2})(f_k - \delta)}{\{(1 - \tau)(1 - s_{L2})(f_k - \delta)\}^2} \{s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W)k f_{kk} \\ + (s_{L1} - s_W) u f_{uk} + (1 - \tau)(1 - s_{L2}) u f_{uk} - (1 - \tau)(1 - s_{L2}) f_{kk} l\}. \quad (31)$$

Since the denominator is positive, it follows that the sign of (31) depends only upon the numerator. Although the numerator is unsigned, we see that $f_k \rightarrow \infty$ for $k \rightarrow 0$ and that the $\dot{k} = 0$ curve meets the k -axis (see (30)). From (31) we also see that the $\dot{k} = 0$ curve is at its maximum level at l^* , the height of the $\dot{k} = 0$ curve at k^* , where, as Figure 1 makes clear, l^* is defined by :

$$l^* = \frac{s_\pi k f_k - (\delta + \alpha + n)k + (s_\pi - s_W)k^* f_{kk} + (s_{L1} - s_W)u f_{lk}}{(1 - \tau)(1 - s_{L2})f_{kk}} \quad (32)$$

Above the $\dot{k} = 0$ curve, k decreases, below it, k increases.

As Figure 1 shows, unrealized capital gains tax on land shifts the $\dot{k} = 0$ curve upwards, and $\dot{l} = 0$ curve downwards, the consequence is that the equilibrium value of capital-labor ratio increases, but the effect on the equilibrium price of land is ambiguous, i. e.

$$dk/d\tau > 0, dl/d\tau ?$$

The above results are verified from (20) and (25).

Consider now the economic implications of the above results. The consumption out of capital gains, $(1 - s_{L2})\dot{P}L$ leads to a decrease in investment, $(\dot{K} + \delta K)$. Since a capital gains tax on land implies a decrease in the consumption out of capital gains, it leads to the increase in the equilibrium capital-labor ratio. The increase in the equilibrium capital-labor ratio increases the rental rate of land and decreases the net rate of return on capital, and in doing so, has the effect of raising the equilibrium price of land. But at the same time, this effect is partially offset by the decrease in the expected capital gains of landowners due to a tax on unrealized capital gains. Thus, the effect on the equilibrium price of land is ambiguous.

3.2.2. In the case where the revenues out of the capital gains tax on land are spent on investment

Let us examine when the revenues are spent on investment. The $\dot{l} = 0$ curve is unchanged, it only alter the $\dot{k} = 0$ curve. Denote the rate of government savings as s_G . Assuming that government savings are directed to investment, and considering (25), we get

$$\begin{aligned} \dot{k} = & s_{\pi} k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u + (1 - s_{L2}) (1 - \tau) u f_u \\ & - (1 - s_{L2}) (1 - \tau) (f_k - \delta) l - s_G \tau u f_u + s_G \tau (f_k - \delta) l - (\delta + \alpha + n) k. \end{aligned} \quad (25)'$$

Letting $s_G = 1$, we obtain

$$\begin{aligned} \dot{k} = & s_{\pi} k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u + (1 - s_{L2}) (1 - \tau) u f_u \\ & - (1 - s_{L2}) (1 - \tau) (f_k - \delta) l - \tau u f_u + \tau (f_k - \delta) l - (\delta + \alpha + n) k. \end{aligned} \quad (25)''$$

When the revenues are spent on investment, the dynamic behavior of the system is thus described by (20) and (25)'.

From (25), we get the $\dot{k} = 0$ curve :

$$l = \frac{s_{\pi} k f_k - (\delta + \alpha + n) k + s_W (f - k f_k - u f_u) + s_{L1} u f_u + \{(1 - \tau)(1 - s_{L2}) - \tau\} u f_u}{\{(1 - \tau)(1 - s_{L2}) - \tau\} (f_k - \delta)} \quad (30)''$$

Differentiating (30)'' with respect to k , and assuming $s_{\pi} = s_{L1}$ for analytical simplicity, we derive the formula for the slope of the $\dot{k} = 0$ curve :

$$\begin{aligned} l_k(k)|_{\dot{k}=0} = & \frac{\{(1 - \tau)(1 - s_{L2}) - \tau\} (f_k - \delta)}{[\{(1 - \tau)(1 - s_{L2}) - \tau\} (f_k - \delta)]^2} \{s_{\pi} f_k - (\delta + \alpha + n) + (s_{\pi} - s_W) \\ & (k f_{kk} - u f_{uk})\} + \{(1 - \tau)(1 - s_{L2}) - \tau\} u f_{uk} - \{(1 - \tau)(1 - s_{L2}) - \tau\} f_{kk} \\ & \{s_{\pi} k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u + (1 - s_{L2}) (1 - \tau) u f_u\}. \end{aligned} \quad (31)''$$

Since the denominator is positive, it follows that the sign of (31)'' depends only upon the numerator. Although the numerator is unsigned, we see that $f_k \rightarrow \infty$ for $k \rightarrow 0$ and that the $\dot{k} = 0$ curve meets the k -axis (see (30)'').

From (31)'' we also see that the $\dot{k} = 0$ curve is at its maximum level at l^{***} , the height of the $\dot{k} = 0$ curve at k^{**} , where, as Figure 1 makes clear, l^{***} is defined by :

$$l^{***} = \frac{s_{\pi} k f_k - (\delta + \alpha + n) k + (s_{\pi} - s_W) k f_{kk} + (s_{L1} - s_W) u f_{uk}}{\{(1 - \tau)(1 - s_{L2}) - \tau\} (f_k - \delta) f_{kk}}. \quad (32)''$$

Above the $\dot{k} = 0$ curve, k decreases, below it, k increases.

From (30)', (31)' and (32)', when we assume the revenues are spent on investment and that $\{(1-\tau)(1-s_{L2})-\tau\}$ is positive, the $\dot{k} = 0$ curve is shifted up even more (while the $\dot{l} = 0$ curve is unchanged), the consequence is that the equilibrium value of capital-labor ratio increases even more. The effect on the equilibrium price of land is ambiguous, i. e.

$$dk/d\tau > 0, dl/d\tau ?$$

The above results are verified from (20)' and (25)'.

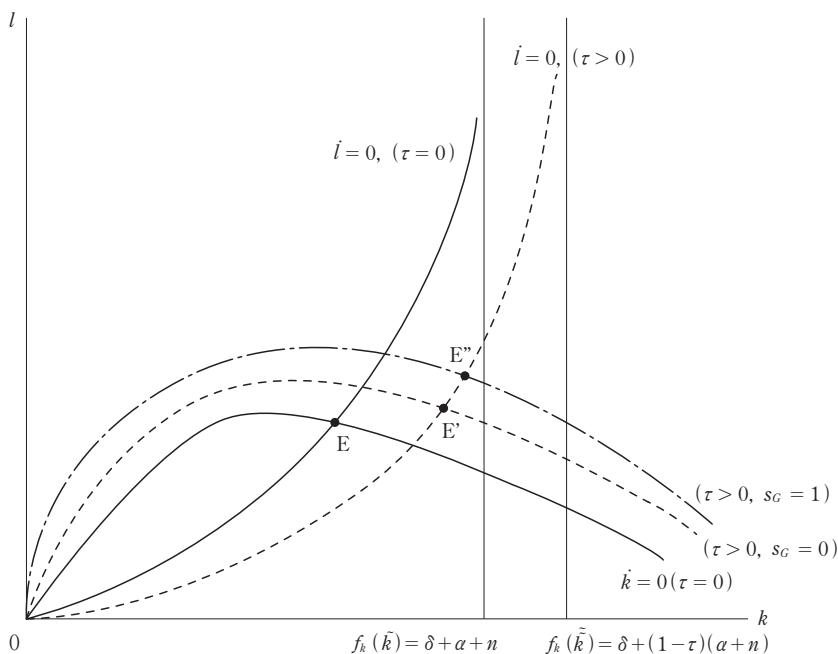


Figure 1. Phase diagram of the system

3.3. Stability of the Equilibrium Growth Path

We shall now examine the stability of the equilibrium growth path. In the previous paper (Aono (2016)), we examined the stability of the long-run steady state path and investigated the paths which do not converge to the long-run steady state in greater detail when there is no unrealized capital gains tax on land. As long as we assume that expected and actual price changes are identical, i. e. $\dot{P}^e/P = \dot{P}/P$, the following result has been showed. When landowners seek capital gains on land and consume out of the capital gains, the equilibrium point is a saddle point, and the dynamic behavior of the system has been illustrated in a phase diagram, (see Figure 1 in the previous paper (Aono (2016))). Although unrealized capital gains tax on land shifts the $\dot{k} = 0$ curve upwards, and $\dot{l} = 0$ curve downwards, it does not change the stability properties of the equilibrium growth path. i. e., the equilibrium point (k^*, l^*) is a saddle point.

In the following, we shall show that the equilibrium point (k^*, l^*) is locally a saddle point when the revenues out of the capital gains tax on land are entirely spent on consumption, or on investment.

It is sufficient to prove that

$$\det. J^* < 0, \tag{33}$$

$$J^* = \begin{bmatrix} J_{11}^* & J_{12}^* \\ J_{21}^* & J_{22}^* \end{bmatrix}$$

where

$$\begin{aligned} J_{11}^* &= s_\pi f_k - (\delta + \alpha + n) + k^* f_{kk} (s_\pi - s_W) + u f_{uk} (s_{L1} - s_W) \\ &\quad + \{(1 - s_{L2})(1 - \tau) - s_G \tau\} (u f_{uk} - l^* f_{kk}), \\ J_{12}^* &= \{s_G \tau - (1 - s_{L2})(1 - \tau)\} (f_k - \delta), \\ J_{21}^* &= \frac{-1}{(1 - \tau)} (u f_{uk} - l^* f_{kk}), \end{aligned}$$

$$J_{22}^* = \frac{1}{(1-\tau)} \{ (f_k - \delta) - (1-\tau)(\alpha+n) \}.$$

J^* is the Jacobian of the system (20) and (25) evaluated at the equilibrium point (k^*, l^*) .

It is verified that

$$\begin{aligned} \det. J^* &= \frac{1}{(1-\tau)} \{ s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W) k^* f_{kk} + (s_{L1} - s_W) u f_{uk} \} \\ &\quad \{ (f_k - \delta) - (1-\tau)(\alpha+n) \} - \{ (1-s_{L2})(1-\tau) - s_G \tau \} \\ &\quad (u f_{uk} - l^* f_{kk}) (\alpha+n). \end{aligned} \quad (34)$$

Consequently,

$$\det. J^* < 0$$

with $0 < s_W \leq s_{L1} = s_\pi \leq s_{L2} < 1$, $0 \leq s_G \leq 1$.

Thus we have shown that the equilibrium point (k^*, l^*) is locally a saddle point under the above assumption.

Letting $s_G = 0$ in (34), we obtain

$$\begin{aligned} \det. J^* &= \frac{1}{(1-\tau)} \{ s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W) k^* f_{kk} + (s_{L1} - s_W) u f_{uk} \} \\ &\quad \{ (f_k - \delta) - (1-\tau)(\alpha+n) \} - \{ (1-s_{L2})(1-\tau) \} (u f_{uk} - l^* f_{kk}) \\ &\quad (\alpha+n) < 0. \end{aligned} \quad (34)'$$

(34)' shows that the equilibrium point is locally a saddle point when the revenues out of the capital gains tax on land are entirely spent on consumption.

Letting $s_G = 1$ in (34), we obtain

$$\begin{aligned} \det. J^* &= \frac{1}{(1-\tau)} \{ s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W) k^* f_{kk} + (s_{L1} - s_W) u f_{uk} \} \\ &\quad \{ (f_k - \delta) - (1-\tau)(\alpha+n) \} - \{ (1-s_{L2})(1-\tau) - \tau \} (u f_{uk} - l^* f_{kk}) \\ &\quad (\alpha+n) < 0. \end{aligned} \quad (34)''$$

(34)" shows that the equilibrium point is locally a saddle point when the revenues out of the capital gains tax on land are entirely spent on investment.

4. Effects of the increase in wealth on inequality and wealth-income ratio when unrealized capital gains tax is imposed

4.1. Increase in wealth per capita and rising inequality

Let us examine whether the increase in wealth increases inequality or not. For this purpose, we shall examine wealth per capita, and the wealth-income ratio on the steady growth path when unrealized capital gains tax is imposed.

For simplicity, we assume that wealth consists of two components, capital and land. The assumption is not so unrealistic as it might seem. Stiglitz (2015a) asserts that "The most important source of the disparity between the growth of wealth and the growth of productive capital is land: much of the increase in wealth is an increase in the value of land—not associated with any increase in the amount of land" (pp. 2).

Letting w denote wealth per capita, and considering (26), we get

$$\frac{dw}{dk} = \frac{d}{dk}(k+l) = 1 + \frac{[uf_{uk}\{(f_k - \delta) - (1-\tau)(\alpha+n)\} - f_{kk}uf_u]}{\{(f_k - \delta) - (1-\tau)(\alpha+n)\}^2} > 1. \quad (35)$$

Whether landowners seek capital gains on land or not, w increases more than k . Furthermore, from equation (35), when they seek capital gains on land, w increases much more than k . This is because when landowners seek capital gains on land, the denominator, $\{(f_k - \delta) - (1-\tau)(\alpha+n)\}^2$ of the right hand side of (35) becomes smaller and dominant.

Let us compare the wealth per capita, w , along the steady growth path when unrealized capital gains tax is imposed with the wealth per capita when there is no capital gains tax ($\tau = 0$). Since the denominator, $\{(f_k - \delta) - (1-\tau)(\alpha+n)\}^2$ of the

right hand side of (34) becomes smaller and dominant, if $\alpha + n > (-df_k/d\tau)$, it is plausible that the wealth per capita without capital gains tax ($\tau = 0$) increases more than the wealth per capita with unrealized capital gains tax on land ($\tau > 0$). In other words, if $\alpha + n > (-df_k/d\tau)$, it is plausible that unrealized capital gains tax on land decreases the disparity of the wealth per capita.

4.2. Wealth-income ratio on the steady growth path

Let us examine the wealth-income ratio along the steady growth path. Denoting γ as the wealth-income ratio along the steady growth path, γ is given by

$$\gamma = \frac{W}{Y} = \frac{K+PL}{Y} = \frac{w}{f(k)} = \frac{k+l}{f(k)}. \quad (36)$$

Differentiating (36) with respect to k , we obtain

$$\begin{aligned} \frac{d\gamma}{dk} &= \frac{d}{dk} \left(\frac{k+l}{f(k)} \right) = \frac{\frac{dw}{dk} f(k) - f_k (k+l)}{f(k)^2} \\ &= \frac{1}{f(k)^2} \left[\left[1 + \frac{uf_{uk} \{ (f_k - \delta) - (1-\tau)(\alpha+n) \} - f_{kk} uf_u}{\{ (f_k - \delta) - (1-\tau)(\alpha+n) \}^2} \right] f(k) \right. \\ &\quad \left. - f_k \left\{ k + \frac{uf_u}{(f_k - \delta) - (1-\tau)(\alpha+n)} \right\} \right]. \end{aligned} \quad (37)$$

Since $dw/dk > 1$, as long as $Y \geq (r+\delta)W$, γ increases more than k . Furthermore, from (37), when landowners seek capital gains on land, w increases much more than k .

Let us compare the wealth-income ratio, γ , along the steady growth path when unrealized capital gains tax is imposed with the wealth-income ratio when there is no capital gains tax ($\tau = 0$). Since the denominator, $\{ (f_k - \delta) - (1-\tau)(\alpha+n) \}^2$ of the right hand side of (35) becomes smaller and dominant, if $\alpha + n > (-df_k/d\tau)$, it is plausible that the wealth-income ratio without capital gains tax ($\tau = 0$) increases more than the wealth per capita with unrealized capital gains tax on land ($\tau > 0$).

In other words, if $\alpha + n > (-df_k/d\tau)$, it is plausible that unrealized capital gains tax on land decreases the disparity of the wealth-income ratio.

5. Concluding Remarks

In this paper, we presented a neoclassical economic growth model with unrealized capital gains tax on land, and examined the characteristics and the stability of the long-run steady state paths. We have shown the following results. First, unrealized capital gains tax on land increases the equilibrium value of capital-labor ratio, but the effect on the equilibrium price of land is ambiguous. The economic implications of the above results are as follows. Since a capital gains tax on land implies a decrease in the consumption out of capital gains, it leads to the increase in the equilibrium capital-labor ratio. The increase in the equilibrium capital-labor ratio increases the rental rate of land and decreases the net rate of return on capital, and in doing so, has the effect of raising the equilibrium price of land. But at the same time, this is partially offset by the decrease in the expected capital gains of landowners due to a tax on unrealized capital gains, and in doing so, has the effect of reducing the equilibrium price of land. Thus, the effect on the equilibrium price of land is ambiguous.

Let us compare the equilibrium value of capital-labor ratio when the revenues out of the capital gains tax on land are entirely spent on consumption with the equilibrium capital-labor ratio when the revenues out of the capital gains tax on land are entirely spent on investment. It is shown that the equilibrium capital-labor ratio of the latter is higher than that of the former.

Second, as long as we assume that expected and actual price changes are identical, unrealized capital gains tax on land does not change the stability properties of the equilibrium growth path, i. e., the equilibrium point is a saddle point. The reason is as follows. Although unrealized capital gains tax on land has the effect of

decreasing the expected capital gains on land, we assume that expected and actual price changes are identical. Therefore, the increase in the actual rate of land prices raises the expected rate of land price, and in doing so, raises the expected capital gains on land. Thus, as long as we assume that expected and actual price changes are identical, unrealized capital gains tax on land does not change the stability properties of the equilibrium growth path.

Third, comparing the wealth per capita and the wealth-income ratio along the steady growth path when unrealized capital gains tax is imposed with the wealth per capita and the wealth-income ratio when there is no capital gains tax, it is plausible that unrealized capital gains tax on land decreases the disparity of the wealth per capita and the wealth-income ratio.

Two extensions of our model are being considered. First, we have examined the effects of unrealized capital gains taxation on land. However, most countries have opted for a realization-based capital gains tax on land. Perhaps significant arguments against unrealized capital gains taxation on land are that unrealized capital gains taxation on land makes it necessary to evaluate unrealized capital gains on land and that it would be extremely difficult to evaluate the unrealized increment of land value. From the practical point of view, instead of imposing unrealized capital gains tax on land, it is necessary to propose a new method of taxation on land. Proposing a new method of taxation on land is left for future research.

Second, in this paper, we assumed that expected and actual price changes are identical. The result is that unrealized capital gains tax on land does change the stability properties of the equilibrium growth path, i. e. the equilibrium point is a saddle point. What happens if we assume that expected and actual price changes are not identical? This is an interesting research topic for the future.

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