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## 1. Introduction

In this paper we present the model in the case where landowners do not seek capital gains on land, but seek rents. We compare the above the model in this paper with the model when landowners seek capital gains on land discussed in Aono (2016a) and (2016b). And doing so, we make the matter clear arising from the behavior of landowners who seek capital gains on land.

The rest of the paper is organized as follows. Section 2 outlines the basic structure of the model, and derives the dynamic equation of the economy. Section 3 examines the uniqueness and stability of the positive equilibrium, and compares the steady state capital-labor ratio in the economy of capital gains seeking landowners with that of non-capital gains seeking land owners. Section 4 examines the wealth-income ratio along the steady state growth path, and compares the steady state wealth-income ratio in the economy of capital gains seeking landowners with that of non-capital gains seeking landowners. Section 5 summarizes the main results of this paper.

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\*\*In a sense, this paper is a revised version of Aono (1976). In this paper we present the model in the case where landowners do not seek capital gains on land, and compare the model in this paper with the model when landowners seek capital gains on land

## 2. Model

### 2. 1. The basic model

The basic model and notation in this section are as follows.

(a) Technology.

Output,  $Y$  is a function of inputs of labor,  $N$ , capital,  $K$ , and land,  $L$ . The production function for this output is twice differential, homogeneous of degree one, in addition, it has the following properties: (i) positive and diminishing marginal productivities of the three factors, (ii) impossibility of any product without one of the factors. Technical change is assumed to be land-augmenting and labor-augmenting. The rate of land-augmenting technical progress and the rate of labor-augmenting technical progress are denoted as  $h$  and  $\alpha$ , respectively. The above assumptions are summarized by

$$Y = F(K, N^*, L^*), F_K, F_{N^*}, F_{L^*} > 0, F_{KK}, F_{N^*N^*}, F_{L^*L^*} < 0, \quad (1)$$

$$L^* = L e^{ht}, \quad (2)$$

$$N^* = N e^{\alpha t}. \quad (3)$$

(b) The supply of land and labor.

Land is assumed to be fixed in supply, and labor is assumed to grow at the given exponential rate,  $n$ . Thus the sum of the rate of labor-augmenting technical progress and the rate of growth of labor is  $\alpha + n$ . We assume that the following conditions are satisfied.

$$L = \bar{L}, \bar{L} : \text{constant} \quad (4)$$

$$N = N_0 e^{nt}, \quad (5)$$

$$0 < \alpha + n < 1. \quad (6)$$

## (c) Land asset market

Let us consider a portfolio balance equation. Since we assume that landowners do not seek capital gains on land, we obtain the following portfolio balance equation ;

$$\rho^e = r^e P. \quad (7)$$

where  $P$  is the price of land in terms of goods,  $\rho^e$  is the expected rent of land, and  $r^e$  is the expected net rate of return on capital. The equation (7) tells us that the land asset market is in equilibrium when the expected rent-price ratio,  $\rho^e/P$  is equal to the expected net rate of return on capital,  $r^e$ .

For simplicity, we assume that the expected and the actual rent of land are identical, and that the expected and the actual net rate of return on capital are identical, i. e.

$$\rho^e = \rho, \quad r^e = r. \quad (8)$$

## (d) Consumption function and savings function

We now consider the consumption function, and hence the savings function. We assume a generalized Cambridge savings function instead of a proportional savings function. Since profits, wages and rents are different characteristic, we assume that the propensity to save out of profits, wages and rents are not always equal, and that the propensity to save out of profits and rents are larger than the propensity to save out of wages. When there is an increase in output, landowners are able to increase rents by virtue of their ownership without doing anything else. Since we assume that landowners do not consume out of capital gains on land, we get the following consumption function ;

$$C = Y - \{s_\pi (r + \delta)K + s_W RN + s_{L1}\rho L\}$$

$$\text{with } 0 \leq s_W < s_{L1}, s_\pi \leq 1, \quad (9)$$

where  $s_\pi$  : the saving rate of capitalists (entrepreneurs) out of profits,  $s_W$  : the saving rate of workers,  $s_{L1}$  : the saving rate of landowners out of rents,  $\delta$  : the rate of depreciated capital,  $r$  : the net rate of return on capital,  $R$  : the real wage rate,  $\rho$  : the rental rate of land.

Since total output,  $Y$  is equal to consumption,  $C$  plus capital depreciation,  $\delta K$ , plus net investment,  $\dot{K}$ , we get

$$Y = C + \delta K + \dot{K}. \quad (10)$$

Substituting (9) into (10) yields

$$\dot{K} + \delta K = \{s_\pi (r + \delta)K + s_W RN + s_{L1}\rho L\}$$

$$\text{with } 0 \leq s_W < s_{L1}, s_\pi \leq 1. \quad (11)$$

#### (e) Determination of factor prices

Given  $R$ ,  $\rho$  and the production function which is homogeneous of degree one (exhibits constant returns to scale), competitive producers can maximize the net rate of return on capital when the marginal productivities of capital,  $F_K$ , labor,  $F_N$  and land,  $F_L$  are equal to  $r + \delta$ ,  $R$ , and  $\rho$ , respectively. Thus, we get

$$F_K = r + \delta, \quad (12)$$

$$F_N = R, \quad (13)$$

$$F_L = \rho. \quad (14)$$

## 2.2. Derivation of dynamic equation

It turns out that the dynamics of economy can best be described in terms of the ratios,  $l \equiv PL/N^*$ ,  $u \equiv L^*/N^*$  and  $k \equiv K/N^*$ .

From the equation (1), we obtain

$$\frac{Y}{N^*} = F(K/N^*, 1, L^*/N^*).$$

Thus, letting  $y = Y/N^*$ , the production function can be rewritten as a function of  $k$  and  $u$ .

$$\frac{Y}{N^*} = f(k, u). \quad (15)$$

Differentiating the above equation with respect to  $K$ ,  $N^*$  and  $L^*$ , we get

$$F_K = f_k(k, u), \quad (16)$$

$$F_{N^*} = f(k, u) - kf_k(k, u) - uf_u(k, u), \quad (17)$$

$$F_{L^*} = f_u(k, u). \quad (18)$$

For analytical simplicity, we shall assume that  $u$  is constant. Since  $u = L^*/N^*$ , the assumption implies that technical change is of such a character that the rate of land-augmenting technical progress is equal to the sum of the rate of growth of labor supply and the rate of labor-augmenting technical progress. When output,  $Y$  is a function of inputs of labor,  $N$ , and capital,  $K$ , it is well known from the standard growth theory that a steady state requires that technical progress be pure labor-augmenting. By the same token, if land supply is fixed, the rate of growth of the rental rate of land must be equal to the rate of land-augmenting technical progress on the steady state growth path (see Aono (1976) and Stiglitz (2015)).

The above assumption may not be so coincidental, once we introduce a theory of endogenous factor bias. Stiglitz (2014b) showed that the bias is determined by relative shares, and if the elasticity of substitution is less than one, as land becomes

scarce, there are greater incentives for land-augmenting technical progress. Thus,  $F_K$ ,  $F_{N^*}$  and  $F_{L^*}$  can be written as a function of  $k$ .

Differentiating  $k \equiv K/N^*$  with respect to time  $t$ , and using (3), (4) and (5), we get

$$\hat{k} = \dot{K} - (\alpha + n), \quad (19)$$

where  $\hat{k} = \dot{k}/k$ ,  $\dot{k} = dk/dt$ .

Using (2), (3), (12), (13) and (14), the equation (11) can be rewritten as

$$\begin{aligned} \dot{K} + \delta &= s_\pi F_K + s_W \left( \frac{F_{N^*}}{k} \right) + s_{L1} F_{L^*} \frac{u}{k}, \\ \text{with } 0 &\leq s_W < s_{L1}, \quad s_\pi \leq 1. \end{aligned} \quad (20)$$

Differentiating  $l \equiv PL/N^*$  with respect to time  $t$ , we get

$$\hat{l} = \hat{P} + \hat{L} - \hat{N}^*, \quad (21)$$

where  $\hat{l} = \dot{l}/l$ ,  $\dot{l} = dl/dt$ .

The consumption function is

$$\begin{aligned} C &= Y - \{s_\pi (r + \delta) K + s_W RN + s_{L1} \rho L\} \\ \text{with } 0 &\leq s_W < s_{L1}, \quad s_\pi \leq 1, \end{aligned} \quad (9)$$

Using the definition,  $l \equiv PL/N^*$ , and using the results that  $F_L = f_u e^{ht}$ ,  $L/N^* = u e^{-ht}$ , and  $r = f_k - \delta$ , from (7) and (8) we get

$$l = \frac{u f_u}{(f_k - \delta)}. \quad (22)$$

Substituting (16), (17) and (18) into (20), we obtain

$$H(k) \equiv \dot{k} = s_\pi k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u - (\delta + \alpha + n) k.$$

$$\text{with } 0 \leq s_W < s_{L1}, \quad s_\pi \leq 1. \quad (23)$$

The equation (23) is the dynamic equation which describes the economy in which landowners do not seek capital gains on land.

### 3. Characteristics of the equilibrium growth path

#### 3.1. The uniqueness and stability of the positive equilibrium

We shall show the uniqueness and stability of the nonzero equilibrium. It is sufficient to show that

$$H_k(k) \big|_{H(k)=0} < 0.$$

For this purpose, we differentiate (23) with respect to  $k$ , and we have

$$H_k(k) = s_\pi f_k - (\delta + \alpha + n) + k f_{kk} (s_\pi - s_W) + u f_{uk} (s_{L1} - s_W), \quad (24)$$

where  $f_{kk}$  and  $f_{uk}$  are the respective second-order partial derivatives of  $f(k)$ . By the assumption of diminishing marginal productivity,  $f_{kk}$  is negative. Under the further assumption that the factors are cooperative,  $f_{uk}$  is positive. For analytical simplicity, we assume that the propensity to save out of profits,  $s_\pi$  is equal to the propensity to save out of rents,  $s_{L1}$ . Then, (24) can be rewritten as

$$\begin{aligned} H_k(k) &= s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W) (k f_{kk} + u f_{uk}) \\ &= s_\pi f_k - (\delta + \alpha + n) - (s_\pi - s_W) N^* F_{N^*K} \end{aligned}$$

$$\text{with } 0 \leq s_W \leq s_{L1} = s_\pi \leq 1. \quad (25)$$

Since  $F_{N^*K}$  is positive, and since from (23), it is seen that  $s_\pi f_k$  is less than  $\delta + \alpha + n$  at the positive equilibrium point, it follows that (23) evaluated at the positive equilibrium point  $k^{**} > 0$  is negative, that is,



$$H_k(k)|_{H(k)=0} < 0.$$

The qualitative behavior of the system is illustrated in Figure 1. The movement in the variable  $k$  is indicated by an arrow. If  $k$  is larger (smaller) than  $k^{**}$ ,  $k$  decreases (increases). The arrow intersects at the equilibrium point  $k^{**} > 0$ . It is clear from Figure 1 that the unique equilibrium point  $k^{**} > 0$ , if it exists, is stable.

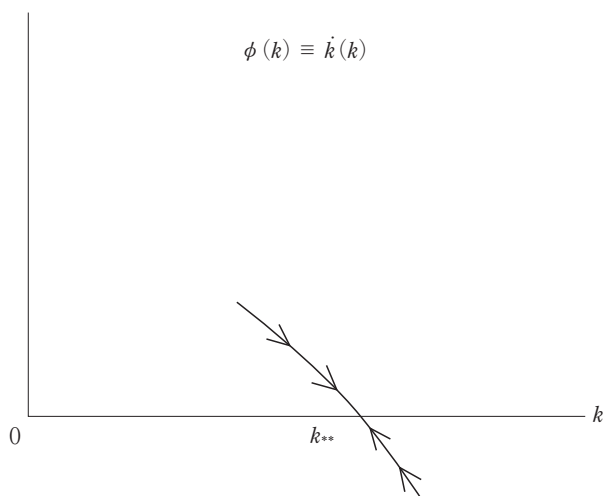


Figure 1

Thus we have proved the uniqueness and stability of the positive equilibrium point  $k^{**} > 0$  under the assumption that the propensity to save out of profits,  $s_\pi$  is equal to the propensity to save out of rents,  $s_{L1}$ .

It is interesting to notice that if land is fixed in supply, the steady state capital-labor ratio  $k^{**}$  in the economy where landowners do not seek capital gains on land is higher than the steady state capital-labor ratio  $k^*$  in the economy where landowners seek capital gains on land. Since  $k^{**}$  is the root of (26) and  $k^*$  is the root of (26)',

$$s_\pi k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u - (\delta + \alpha + n) k = 0, \quad (26)$$

$$s_\pi k f_k + s_W (f - k f_k - u f_u) + s_{L1} u f_u + (1 - s_{L2}) u f_u - (1 - s_{L2}) (f_k - \delta) l - (\delta + \alpha + n) k = 0. \quad (26)'$$

It is easily proved from (26) and (26)' that

$$k_{**} > k^*. \quad (27)$$

We see from (7), (8), (12), (14), (16), (18), and using the definition,  $l \equiv PL/N^*$  and the results,  $F_L = f_u e^{ht}$ ,  $L/N^* = u e^{-ht}$ , the steady state of land-labor ratio  $l_{**}$ , and the steady state of the price of land are :

$$l_{**} = \frac{u f_u(k_{**})}{f_k(k_{**}) - \delta}, \text{ and } P(t) = \frac{\rho(t)}{r_{**}} = \frac{\rho(t)}{f_k(k_{**}) - \delta}. \quad (28)$$

Consider now the economic implications of the above results. Our results crucially depend on the two assumptions ; the portfolio balance equation and the savings function. If landowners do not seek capital gains on land, the land market is in equilibrium when the rent-price ratio  $\rho/P$  is equal to the net rate of return on capital  $r$ . For the land market to be in equilibrium, the net rate of return on capital,  $r = f_k - \delta$  does not have to exceed the rate of growth of output,  $\alpha + n$ . It is possible that more capital is accumulated and that the net rate of return on capital falls to the point where  $r = \alpha + n$  as long as  $r$  is positive.

#### 4. Wealth-income ratio on the steady growth path

In our model, wealth  $W$  consists of capital  $K$  and land  $L$ , that is, at time  $t$ ,  $W(t)$  is given by

$$W(t) = K(t) + P(t)L \quad (29)$$

Letting  $w$  denote wealth per capita, and considering (28), we get

$$\frac{dw}{dk} = \frac{d}{dk}(k+l) = 1 + \frac{uf_{uk}\{f_k - (\delta)\} - f_{kk}uf_u}{(\{f_k - (\delta)\})^2} > 1. \quad (30)$$

The equation (30) shows that  $w$  increases more than  $k$ .

When landowners seek capital gains on land, It has been shown by Aono (2015) that the following equation holds.

$$\frac{dw}{dk} = \frac{d}{dk}(k+l) = 1 + \frac{uf_{uk}\{f_k - (\delta + \alpha + n)\} - f_{kk}uf_u}{(\{f_k - (\delta + \alpha + n)\})^2} > 1. \quad (30)'$$

Comparing (30) with (30)', we can say as follows. Whether landowners seek capital gains on land or not,  $w$  increases more than  $k$ . When landowners seek capital gains on land,  $w$  increases much more than  $k$ . This is because when landowners seek capital gains on land, the denominator,  $\{f_k - (\delta + \alpha + n)\}^2$  of the right hand side of (30)' becomes smaller and dominant.

Let us examine the wealth-income ratio along the steady growth path. Denoting  $\gamma$  as the wealth-income ratio,  $\gamma$  is given by

$$\gamma = \frac{W}{Y} = \frac{K+PL}{Y} = \frac{w}{f(k)} = \frac{k+l}{f(k)}. \quad (31)$$

Differentiating (31) with respect to  $k$ , we obtain

$$\frac{d\gamma}{dk} = \frac{d}{dk}\left(\frac{k+l}{f(k)}\right) = \frac{\frac{dw}{dk}f(k) - f_k(k+l)}{f(k)^2}. \quad (32)$$

Since  $dw/dk > 1$ , as long as  $Y \geq (r+\delta)W$ ,  $\gamma$  increases more than  $k$ .

Furthermore, from (30) and (30)', when landowners seek capital gains on land,  $w$  increases much more than  $k$ . Therefore, when landowners seek capital gains on land, as long as  $Y \geq (r + \delta)W$ ,  $\gamma$  increases much more than  $k$ .

## 5. Concluding remarks

In this paper we have present the model in the case where landowners do not seek capital gains on land, and examined the characteristic and stability of the steady state growth path. Let us compare the economy of capital gains seeking landowners with that of non-capital gains seeking landowners. Aono (2015) examined the characteristic of the steady state growth path when landowners seek capital gains on land and consume out of the capital gains. Furthermore, Aono (2016a) examined the stability of the equilibrium path, and considered the paths which do not converge to the long-run steady state in greater detail.

Under the assumption of fixed land supply, the problems resulting from the economy of capital gains seeking landowners may be summarized as follows. First, when landowners seek capital gains on land, the rate of net return on capital (the rate of profits) must exceed the rate of economic growth along the steady state growth path. Second, comparing a capital gains seeking economy with a non-capital gains seeking economy, the steady state price of land in the former is higher than that in the latter. Thus the wealth-income ratio along the steady growth path in the former is higher than that in the latter. If we assume that landowners consume out of capital gains on land, the steady state capital-labor ratio in the former is lower than that in the latter. In this respect, a capital gains seeking economy impedes capital accumulation. Third, when landowners seek capital gains on land and consume out of capital gains on land, the steady state growth path is unstable, that is, as Aono (2016a) has shown, the equilibrium point is a saddle point. On the other hand, if landowners do not seek capital gains on land, the

steady state growth path is stable. In this respect, a capital gains seeking economy contributes to volatility without enhancing long-term economic performance.

We have shown that a capital gains seeking economy impedes capital accumulation, contributes to volatility and rising inequality due to rising land prices.

Two extensions of our model are being considered. First, we did not incorporate link between credit, collateral, land prices and capital accumulation into the growth model. Incorporating these factors into the growth model, and examining the effects of these factors on the growth paths are left for future research. Second, we disregard the effects of the capital gains tax on the steady state price of land, capital accumulation and the stability of the system. The effects of the capital gains tax on a capital gains-seeking economy are an interesting research topic for the future.

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