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Examining Approaches to Bond Management Under Low Interest Rates

Kenji Hamamoto

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Abstract

This study aimed to provide a framework for Japanese local government officials engaged in bond management. Specifically, it examined investment methods that can serve as a guideline for bond investment activities in a low interest rate environment. The results of the study can be summarized as follows. When the yield curve is assumed to remain unchanged, repeated bond replacements can generate gains exceeding the gains from holding them to their maturities because of the roll-down effect.

Keywords : Bond management・Yield curve・Roll down effect
JEL Classification Codes : E43・E58・G12

1. Introduction

According to the Japan Finance Organization for Municipalities (2017), only 36% of Japanese municipal funds are invested in marketable securities. As noted by Sezaki (2015), this suggests a reluctance on the part of Japanese local governments to invest in bonds. Local governments with surplus funds should consider investing these in bonds where there is a guaranteed principal, just like deposits, as well as the prospect of a higher return.

However, interest rates in Japan have remained low over a sustained period, making it difficult for local governments to invest in bonds. In the low interest rate environment, it is not possible to generate a return from interest income on bonds. Therefore, investors need to consider generating income through sales pre-maturity or through the replacement of bonds. In an increasingly complex investment environment, investment managers in bonds need a benchmark for their specific investment approach, without which, they will struggle to implement their policies. Most previous studies on bond analysis have focused on theory. Although there are some studies that analyze bond investment strategies from a practical perspective, such as Homer and Leibowitz (1972) and Yamada (2000), studies that look at specific investment approaches are few. In particular, few studies have calculated returns using actual data from the Japanese market and analyzed specific investment approaches in a low interest rate environment (including negative interest rates).

This paper, therefore, aims to provide constructive and practical guidance on the investment management of bonds, namely whether bonds should be held to maturity or sold pre-maturity, including replacement with other bonds. In addition, the funds managed by local governments are public funds, indicating that they can only be invested in liquid assets with a guaranteed principal. Therefore, in this paper, I focus on fixed-income government bonds which are highly liquid and have a guaranteed principal.

The structure of this paper is as follows. First, in Section 2, I examine the use of replacement, which combines not only the sale of bonds but also the purchase of new bonds. Specifically, with a given nominal coupon and an unchanging yield curve, I consider whether replacing a bond can improve returns compared with holding the bond until maturity. In Section 3, I explore bond replacement further. Specifically, I examine whether bond replacement can be more advantageous than holding bonds to maturity if I exclude the potential effect of increasing the nominal coupon through new bond purchases and consider only the roll down effect of bonds sold. In Section 4, I consider the replacement under more realistic conditions where both the shape of the yield curve and the nominal coupon are subject to change. Finally, in Section 5, I conclude the discussion.

2. Hold to maturity or replace ?

In this section, I will discuss the use of replacement investment wherein the improvement of the return is aimed not only by the sale of the bond but also in combination with new purchases. Of course, if there is a bond with more favorable terms than the bond currently owned, it is obvious that replacing the bond with a new one will improve earnings. However, I will examine whether replacing the government bond with another government bond, which is the assumption in this study, will improve earnings over the remaining life of the bond currently held.

2.1 Conditions for replacement

Now, let us assume that we own a bond with a face value of $\overline{A}$, a coupon rate of $\overline{c}_1$, a ownership period of the bond from the time of purchase to the time of sale at a point pre-maturity of h, a remaining period from the time of sale up until maturity of n, and a purchase price of $\overline{P}_{B,1}$; we can sell this bond (hereinafter referred to as the sale bond) for a sale price of $P_{S,1}$. Simultaneously, if we assume that we also purchase a bond with a face value of $\overline{A}$ and a coupon rate of c_2 (hereafter referred to as the newly purchased bond), we need to examine whether we can earn more by replacing this bond than by holding the sale bond until maturity. For simplification of the analysis, it is assumed that there is no accrued interest and that there are no price fluctuations. If a bar (−) is shown above the variable, this indicates that the value is fixed. In addition, The face value and nominal coupon of the bond are constant.

In Figure 1, the return $\pi_{H,1}$ obtained by holding the sale bond (without selling it) for $h+n$ years until maturity is as follows:

$$\pi_{H,1}=(\overline{A}-\overline{P}_{B,1})+\overline{c}_1\overline{A}(h+n) \tag{1}$$

Conversely, if the sale bond is sold after h years of holding, the return $\pi_{S,1}$ over period h is as follows:

$$\pi_{S,1}=(P_{S,1}-\overline{P}_{B,1})+\overline{c}_1\overline{A}h \tag{2}$$

Additionally, the return $\pi_{S,2}$ obtained from the newly purchased bond during the remaining n years of the sale bond can be expressed as follows:

$$\pi_{S,2}=c_2\overline{A}n \tag{3}$$

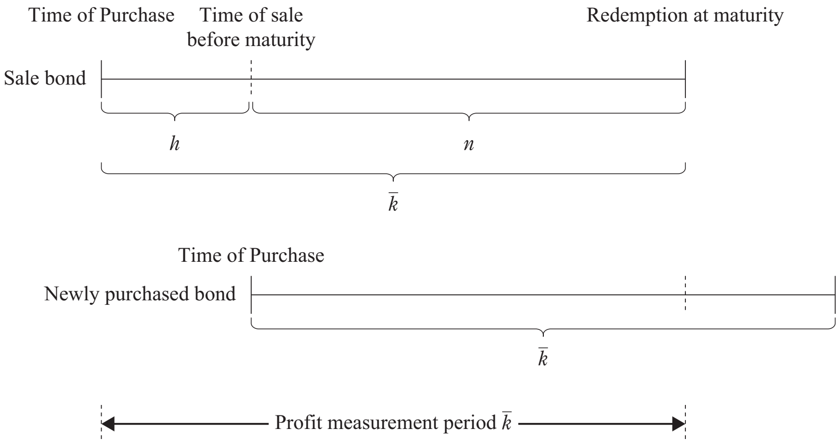
It is assumed here that the bond is replaced only once and that the newly purchased bond does not mature in n years (hence, there is neither gain on the sale nor gain on redemption of the newly purchased bond). A visualization of the replacement is shown in Figure 1.

For it to be more advantageous to replace the sale bond than to hold it until maturity, the sum of Equations (2) and (3) must be greater than that of Equation (1):

$$(c_2-\overline{c}_1)\overline{A}n>\overline{A}-P_{S,1} \tag{4}$$

Equation (4) indicates that the difference in interest income must be greater than the difference between the face value and the sale price in n years and that this condition depends on the sale price of the sale bond, $P_{S,1}$, the remaining term n, and the coupon rate of the newly purchased bond c_2.

Figure 1 : Visualization of replacement (1)



Source : Prepared by the author

2. 2 Replacement decisions with a given coupon rate and an unchanging yield curve

In Equation (4), the coupon rate of the newly purchased bond is exogenously determined by the level of market interest rates at the time ; thus, it is given as $c_2 = \overline{c}_2$. Assuming that the shape of the yield curve of the sale bond remains unchanged in the future over the investment period, the sale amount is uniquely determined by the remaining period n as $P_{S,1} = P_{S,1}(n)$, and Equation (4) can be calculated.

As an example, let us consider the period in Figure 1 and the figures for the "165th and 166th super-long-term bond (20 years)" in Table 1. I begin my analysis with $\overline{A} =$ ¥100, $\overline{c}_1 = 0.6\%$, $\overline{k}(= h + n) = 19.888$ years, and $\overline{P}_{B,1} =$ ¥98. 941. If the shape of the yield curve remains unchanged from the situation in Table 1, then if $n = 19.636$ years, the bond can be sold for $P_{S,1} =$ ¥99. 154

Table 1: Remaining period to maturity, yield, and price

Name of security	Remaining period to maturity (Years)	Yield (%)	Theoretical price (Yen)
Super-long-term JGB 41	0.370	−0.138	100.273
Super-long-term JGB 42	0.370	−0.148	100.277
Super-long-term JGB 43	0.874	−0.146	100.653
Super-long-term JGB 84	7.129	−0.010	104.350
Super-long-term JGB 85	7.375	0.000	104.425
Super-long-term JGB 113	10.882	0.174	104.588
Super-long-term JGB 114	11.132	0.191	104.500
Super-long-term JGB 115	11.132	0.191	104.500
Super-long-term JGB 116	11.378	0.203	104.461
Super-long-term JGB 156	17.384	0.529	101.176
Super-long-term JGB 163	19.137	0.622	99.604
Super-long-term JGB 164	19.384	0.632	99.418
Super-long-term JGB 165	19.636	0.646	99.154
Super-long-term JGB 166	19.888	0.657	98.941

Source: Yields are the compound yields of 20-year super-long-term government bonds, as published by the Japan Securities Dealers Association (November 5, 2018) in the Reference Statistics for OTC Trading of Government Bonds. The bond price is the theoretical value of the price of a bond with a coupon of 0.6%, calculated on the assumption that the relationship between the yield and the remaining period to maturity (yield curve) is unchanged in future. These are also available from the data for 20-year super-long-term JGBs.

when $h = \overline{k} - n = 19.888$ years $- 19.636$ years $= 0.252$ years after the purchase. Assuming that the bond with $\overline{A} =$ ¥100 and $c_2 = 0.7\%$ is purchased at the same time, Equation (4) shows that the return will be ¥1.118 more if the sale bond is replaced than if it is held until maturity.[1] However, using the figures for the "41th and 166th super-long-term bond (20 years)" in Table 1, I estimate that because the bond is sold for $P_{S,1}$ = ¥100.273 when $h = \overline{k} - n$ = 19.888 years − 0.370 years = 19.518 years after the purchase, the amount by which replacement exceeds that of holding to maturity is reduced to ¥0.310.[2)]

Next, let us determine the timing for replacement that maximizes the increase in earnings. The formula for finding the maximum revenue is as follows :

$$\begin{aligned} &\max \quad (\overline{c}_2 - \overline{c}_1)\overline{A}n - (\overline{A} - P_{S,1}(n)) \\ &s.t. \quad \overline{k} > n \end{aligned} \tag{5}$$

Once the variable n is determined, $P_{S,1}$ is also uniquely determined by the assumption of an invariant yield curve for the sale bond. Using the data in Table 1 and substituting $\overline{c}_2$ = 0.7%, let us solve Equation (5). The optimal solution is obtained by selling the bond held after h = 9.005 years (remaining period n = 10.882 years) for $P_{S,1}$ = ¥104.588 and replacing it with a newly purchased bond, which yields ¥5.676 more for the investor by replacing the sale bond than by holding it until maturity.[3)]

The aforementioned results show that bond replacement can earn more than holding sale bonds until maturity. However, these estimation results were obtained by assuming that the shape of the yield curve of the sale bond remains unchanged in the future and by giving an exogenous arbitrary value for the coupon rate of the newly purchased bonds. What if the bond is to be replaced after h = 9.005 years of ownership and if the shape of the yield curve of the sale bond changes to an inverse yield ? For example, if the sale price of the bond falls to $P_{S,1}$ = ¥98.900,

1) Calculating Equation (2) + Equation (3) − Equation (1), I obtain {(¥99.154 − ¥98.941) + ¥100 × 0.6% × 0.252 years + ¥100 × 0.7% × 19.636 years} − {(¥100 − ¥98.941) + ¥100 × 0.6% × 19.888 years} ≒ ¥1.118. This is equal to {(0.7% − 0.6%) × ¥100 × 19.636 years} − (¥100 − ¥99.154) ≒ ¥1.118, which is the result of the calculation in Equation (4).
2) {(0.7% − 0.6%) × ¥100 × 0.370 years} − (¥100 − ¥100.273) ≒ ¥0.310
3) {(¥104.588 − ¥98.941) + ¥100 × 0.6% × 9.005 years + ¥100 × 0.7% × 10.882 years} − {(¥100 − ¥98.941) + ¥100 × 0.6% × 19.888 years} ≒ ¥5.676.

which is lower than the purchase price, the return would be ¥0.012 lower than holding the bond until maturity.[4)] Similarly, even if the shape of the yield curve does not change, if market interest rates decline and the coupon rate on the newly purchased bond falls to $\overline{c}_2 = 0.1\%$, then replacement of the bond will reduce earnings by ¥0.853.[5)] When planning to increase earnings by replacing bonds rather than holding them to maturity, it should be noted that the future shape of the yield curve and market interest rates will have an influence on returns.

3. Replacement decisions using the roll-down effect

In the previous section, I demonstrated that it is possible to improve returns by replacing bonds, and I found that the timing for replacing bonds that maximizes the increase in returns is after $h = 9.005$ years of holding. However, the results were obtained by setting the coupon rate of the newly purchased bond higher than that of the sale bond ($\overline{c}_1 < \overline{c}_2$). In this section, I exclude the effect of the increase in the coupon rate of the newly purchased bond and consider whether the roll-down effect of the sale bond alone can exceed the gain on holding to maturity within $h = 9.005$ years, when the coupon rate $\overline{c}$ of the newly purchased bond is kept the same as that of the sale bond.[6)]

In the previous section, it was assumed that the replacement is made only once; in this section, I extend the assumption to include any number of replacements. To observe the roll-down effect of the sale bond, the future yield curve of the sale bond is assumed to be constant and the newly purchased bond is

4) {(¥98.900 − ¥98.941) + ¥100 × 0.6% × 9.005 years + ¥100 × 0.7% × 10.882 years} − {(¥100 − ¥98.941) + ¥100 × 0.6% × 19.888 years} ≒ −¥0.012

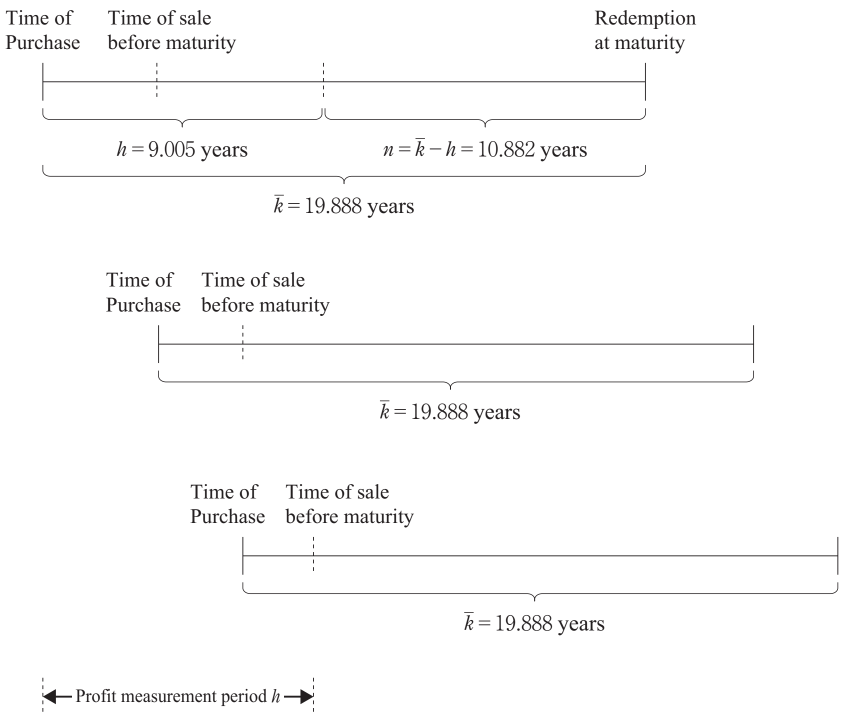
5) {(¥104.588 − ¥98.941) + ¥100 × 0.6% × 9.005 years + ¥100 × 0.1% × 10.882 years} − {(¥100 − ¥98.941) + ¥100 × 0.6% × 19.888 years} ≒ −¥0.853

6) Yamada (2000) is a previous study that analyzed the roll-down effect in the Japanese government bond market.

assumed to follow the same yield curve shape as the sale bond.

The data used are as follows. The reference bond is the bond in Table 1 with $\bar{k} = 19.888$ years remaining to maturity, and when replacing the bond, a bond with this remaining maturity is always purchased, as shown in Figure 2. The timing of replacement is assumed to be the remaining maturity of each 20-year super-long-term bonds in Table 1, meaning that there are 53 possible replacement timings during the holding period of $h = 9.005$ years. The bond may be replaced any number of times during $h = 9.005$ years and may be replaced any number of times

Figure 2 : Visualization of replacement (2)



Source : Prepared by the author

with the same remaining maturity (overlapping combinations of bonds with the same remaining maturity are allowed); however, the period during which there is no bond investment outstanding (the period during which bond investments are interrupted and invested in time deposits, etc.) should be kept as short as possible.

For example, assuming that the bond is held until the remaining life becomes 10.882 years in Table 1 and is replaced (the number of years of holding is 19.888 years − 10.882 years ≒ 9.005 years), then there is only one timing when the bond can be replaced during the 9.005 years after the purchase. If I assume that the bond is held until its remaining life becomes 11.132 years and then replaced (the number of years of holding is 19.888 years − 11.132 years = 8.756 years), there is also only one opportunity to replace the bond during the 9.005 years after purchase (there is no timing for replacement if the bond held for less than 9.005 years − 8.756 years = 0.249 year). Assuming that the bond is held until its remaining life becomes 11.378 years and then replaced (the number of years of holding is 19.888 years − 11.378 years = 8.510 years), during the 9.005 years after purchase, the bond can be replaced one more additional time with another bond with a remaining maturity of 19.636 years (holding period is 19.888 years − 19.636 years = 0.252 year).[7] With this in mind, I can construct a matrix $\mathbf{Q}$ in which the components are the number of replacements q_i^j in the number of years of ownership (timing of replacement) in the i row of the j column combination (Table 2).

Next, under the assumption that the shape of the yield curve is unchanged for both sold and newly purchased bonds, the purchase price of a bond, $\overline{P}_B$, is always the price of the 166th 20-year super-long-term bond in Table 1, ¥98.941; thus, I can create a vertical vector $\boldsymbol{\pi}$ showing the gain on sale — which is $\overline{P}_B$, the purchase price, less P_S, the sale price, or $P_S - \overline{P}_B$ — by years of holding.

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7) Replacement with a bond with a holding period of 9.005 years − 8.510 years = 0.495 year or less is possible.

Table 2 : Combination matrix Q with the number of replacements over time by years of holding

Years of holding \ Combination	①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩	⑪	⑫	⑬	⑭	⑮	…
9.005	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	…
8.756	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	…
8.756	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	…
8.510	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	…
8.510	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	…
8.258	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	…
8.258	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	…
8.258	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	…
8.005	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	…
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1.000	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	…
0.751	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	…
0.504	0	0	0	0	0	1	0	1	0	1	0	0	0	1	0	…
0.252	0	0	0	1	1	0	2	0	2	0	2	0	0	1	3	…

Note : The numbers in the table are the number of replacements. For example, in Column ②, if the owner replaces the bond after 8.756 years of holding, the remaining time to replace it is only 9.005 years − 8.756 years = 0.249 year ; thus, there is no other timing of replacement. In Column ④, if the holder replaces the bond after 8.510 years of ownership, there remains 9.005 years − 8.510 years = 0.495 year, meaning that it can be replaced one more additional time with another bond with a holding period of 0.252 year. Columns ⑫ − ⑮ show that if the bond is replaced after 8.005 years of holding, there remains 9.005 years − 8.005 years = 1.000 year ; thus, either one of the combinations where the bond can be replaced once with a holding period of 1.000 year, once with a holding period of 0.751 year, once with a holding period of 0.252 year after 0.504 year of holding, or three times after 0.252 year of ownership (two times or less is not allowed because the period during which no bond investment is outstanding must be kept as short as possible) each has to be chosen. Note that because the shape of the yield curve is assumed to be constant — for example, the gain on sale is the same for the case of holding and selling after 8.510 years and then holding again for 0.252 year as well as for the case of holding and selling after 0.252 year and then holding again for 8.510 years — only one of these can be used as a candidate for the combination pattern.

Table 3: Gain on sale by years of holding

Years of holding (years) $\mathbf{h}'$	9. 005	8. 756	8. 756	8. 510	…	0. 504	0. 252
Gain on sale (¥) $\boldsymbol{\pi}'$	5. 648	5. 560	5. 560	5. 520	…	0. 477	0. 214

Source: Prepared by the author from Table 1.

Furthermore, for the combination in column j, if $\mathbf{x}$ is a longitudinal vector whose components are the binary variables x^j, which are "1" when adopted and "0" when not adopted, then the total gain on the sale of the bond by repeatedly replacing it during the period of 9. 005 years is $\boldsymbol{\pi}'\mathbf{Qx}$. If the vector of number of years of holding is $\mathbf{h}$, the total duration of holding of the bond is $\mathbf{h}'\mathbf{Qx}$, so the interest income earned over 9. 005 years is $\overline{cA}\,\mathbf{h}'\mathbf{Qx}$.[8] The gain that can be obtained by repeating replacements of the bonds during the 9. 005-year period is $\boldsymbol{\pi}'\mathbf{Qx}+\overline{cA}\,\mathbf{h}'\mathbf{Qx}$. Thus, the maximum gain that can be obtained by combining the optimal timing of replacement over the 9. 005-year period can be obtained by the following equation:

$$
\begin{aligned}
\max \quad & \boldsymbol{\pi}'\mathbf{Qx}+\overline{cA}\,\mathbf{h}'\mathbf{Qx} \\
s.t. \quad & x^j \in \{0,\ 1\} \\
& \mathbf{h}'\mathbf{Qx} \leq h \\
& \sum x^j = 1
\end{aligned} \tag{6}
$$

The first constraint equation in Equation (6) indicates that x^j is a binary variable that takes the value of 0 or 1, the second constraint equation indicates that the total duration of bond holding does not exceed $h = 9.005$ years, and the third constraint equation indicates that there is only one replacement combination to adopt. The

8) Interest income generated during the period $h-\mathbf{h}'\mathbf{Qx}$ when no bond investment is made (e. g., deposit interest if the investment is made in a time deposit) is disregarded.

variable is the binary variable x^j. Using the data in Tables 2 and 3, and substituting $\overline{A} =$ ¥100 and $\overline{c} = 0.6\%$ to solve Equation (6), the optimal solution is obtained by selecting a combination pattern that replaces the bond 17 times after 0.504 year of holding and once after 0.252 year of holding, resulting in a profit of ¥13.622 that can be obtained within 9.005 years.[9] As the return obtained by continuing to hold the bond until maturity is $(\overline{A}-\overline{P}_B)+\overline{cAk} =$ ¥12.992,[10] compared to holding the bond until maturity, the roll-down effect of a repeated replacement of bonds allows the investor to earn up to ¥0.630 more[11] in a shorter period of time of $\mathbf{h}'\mathbf{Qx} = 8.822$ years.[12] As shown above, if the shape of the yield curve in Table 1 remains unchanged in the future, even without an increase in the coupon rate, the roll-down effect alone will allow the bondholders to exceed their holding gains at maturity within $h = 9.005$ years by repeatedly replacing the bonds.

4. Replacement decisions under changing coupon rates and yield curves

In the previous section, I sought to determine how repeated replacements would maximize the increase in earnings due to the roll-down effect. However, to focus on the roll-down effect, the shape of the yield curve and the coupon rate were assumed to be fixed. In this section, I will consider how to make replacement purchases in a more realistic situation wherein the shape of the yield curve, including the coupon rate, changes.

The 163rd super-long-term government bond (20-year) with a coupon rate of

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9) (¥0.477 × 17 times + ¥0.214 × 1 time) + {¥100 × 0.6% × (0.504 year × 17 times + 0.252 year × 1 time)} ≒ ¥13.622.

10) (¥100 − ¥98.941) + ¥100 × 0.6% × 19.888 years ≒ ¥12.992.

11) The sale bond and the new bond are assumed to have the same coupon rate and the same shape yield curve.

12) 0.504 years × 17 times + 0.252 year × 1 time ≒ 8.822 years.

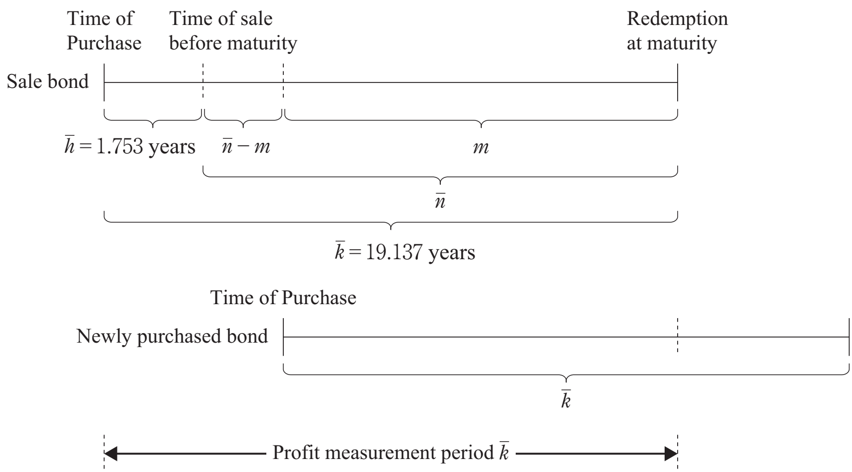
0.6% should yield 0.529% and be priced at ¥101.176 on 640 days after November 5, 2018 (remaining period of 17.384 years), if the yield curve depicted in Table 1 remains constant[13) (the same figures as for the 156th super-long-term bonds in Table 1). However, as of August 6, 2020, 640 days later, the yield is 0.322% and the price is ¥104.69, indicating that the yield has fallen and the price has risen compared to the values assumed from the yield curve in Table 1.

Here, the reference sale bond is the 163rd 20-year super-long-term government bond with a coupon rate of 0.6%, as shown in Table 1, and if I assume that such a bond with a face value of $\overline{A}$ = ¥100, coupon rate $\overline{c}_1 = 0.6\%$, remaining term to maturity $\overline{k} = 19.137$ years and a purchase price of $\overline{P}_B$ = ¥99.604 was sold before maturity at a sale price of $\overline{P}_S$ = ¥104.69, on $\overline{h} = 640$ days (≒1.753 years) after the purchase, then the total of the gain on sale of ¥5.086 and the interest income earned between the purchase and sale of ¥1.052 will be ¥6.138.[14) However, it does not reach the profit of ¥11.878,[15) which can be obtained by holding the bond until maturity; therefore, if the goal is to earn more than the return obtained by holding the bond until maturity, the investor must consider purchasing a new bond after the sale (Figure 3).

If the newly purchased bond has a face value of $\overline{A}$ = ¥100 and a coupon rate of $\overline{c}_2 = 0.3\%$, the interest earned from the newly purchased bond during $\overline{k} - \overline{h}$ years is ¥5.215,[16) which combined with the ¥6.138 from the sale of the bond totals ¥11.353 but does not reach the equivalent return of holding the bond to maturity. However, if the coupon rate of the newly purchased bond is $\overline{c}_2 = 0.4\%$, the total profit from the replacement would be ¥13.091,[17) exceeding the return from holding the bond to maturity. Note, however, that it is not always possible to

13) Note that the theoretical prices in Table 1 are calculated with a coupon rate of 0.6%.
14) (¥104.69 − ¥99.604) + ¥100 × 0.6% × 1.753 years ≒ ¥6.138.
15) (¥100 − ¥99.604) + ¥100 × 0.6% × 19.137 years ≒ ¥11.878.
16) ¥100 × 0.3% × (19.137 years − 1.753 years) ≒ ¥5.215.
17) ¥100 × 0.4% × (19.137 years − 1.753 years) + ¥6.138 ≒ ¥13.091.

Figure 3 : Visualization of replacement (3)



Source : Prepared by the author

purchase a new bond with a coupon rate of 0. 4% when the bond is sold before maturity ; therefore, the investor must consider what is the longest time for the funds to be held in cash deposits after the pre-mature sale and wait for an opportunity to purchase a bond with a coupon rate of 0. 4%.[18)]

The gain of ¥5. 086 on sale can be considered as an advance receipt equivalent to 8. 476 years of interest on the sale bond.[19)] However, if after the sale, the revenue is held for $\bar{n} - m$ = 8. 476 years in cash deposits and the remaining m = 8. 907 years[20)] are used to invest in newly purchased bonds, the gain on replacement investment is only ¥9. 701,[21)] which does not reach the gain on holding to maturity.

18) For simplicity, I omit the interest on deposits and investments other than those held in cash and deposits during the period between the sale and the purchase of the new bond (the period $\bar{n} - m$ in Figure 3).
19) ¥5. 086 / (0. 6% × ¥100) ≒ 8. 476 years.
20) 19. 137 years − 1. 753 years − 8. 476 years ≒ 8. 907 years.

Therefore, the new bond must be purchased before 8.476 years have elapsed after the sale. In other words, if the investment period of the newly purchased bond is m years out of the $\overline{k}-\overline{h}=\overline{n}$ years remaining until the maturity of the sale bond minus the period before the sale, the period m required to earn replacement investment profit that at a minimum exceeds the profit from holding the bond to maturity is obtained by solving Equation (7).

$$\begin{aligned}
&\min \quad \{(\overline{P}_S-\overline{P}_B)+\overline{c}_1\overline{Ah}+\overline{c}_2\overline{Am}\}-\{(\overline{A}-\overline{P}_B)+\overline{c}_1\overline{Ak}\} \\
&s.t. \quad \{(\overline{P}_S-\overline{P}_B)+\overline{c}_1\overline{Ah}+\overline{c}_2\overline{Am}\}>\{(\overline{A}-\overline{P}_B)+\overline{c}_1\overline{Ak}\} \\
&\qquad\quad m>(\overline{k}-\overline{h})-(\overline{P}_S-\overline{P}_B)\,/\,\overline{c}_1\overline{A}
\end{aligned} \tag{7}$$

The first constraint equation in Equation (7) indicates that the gain on replacement investment exceeds the gain on holding to maturity, and the second constraint equation indicates that the investment period of the newly purchased bond is longer than the period $\overline{k}-\overline{h}$ minus the equivalent of the conversion period of advance receipt of the interest on the gain on the sale (8.476 years in this example). The variable is the investment period m of the newly purchased bond. Solving Equation (7), I find that a minimum investment period of $m=14.350$ years is required for the newly purchased bonds.

To summarize the aforementioned details, when the yield curve changes and a gain on sale of ¥5.086 is obtained, a coupon rate of 0.3% is not sufficient for the replacement investment gain to exceed the gain on holding the bond to maturity and a minimum of 0.4% is required. Furthermore, by selling the bond, the investor can receive the equivalent of 8.476 years of interest income in advance; however, if the bond is held in cash deposits for 8.476 years before the new bond is purchased, the replacement investment income will be insufficient. To exceed gain

21) (¥104.69 − ¥99.604) + ¥100 × 0.6% × 1.753 years + ¥100 × 0.4% × 8.907 years ≒ ¥9.701.

on maturity, the new bond must be purchased by 3.033 years after the sale at the latest.[22] In summary, the investor should decide to sell the bond if it expects to be able to purchase a new bond with a coupon rate of 0.4% within 3.033 years after the sale.

5. Conclusion

Although Japanese local governments appear to be reluctant to invest their funds in bonds, they should consider investing in bonds with high rates of return if they have surplus funds. However, interest rates in Japan have remained low, making it difficult for local governments to undertake bond investments.

Therefore, this study examined bond investment methods, with the aim of providing a framework for local government officials who are engaged in bond management in the course of their practice. The main conclusions obtained from the analysis of fixed-income government bonds are as follows. ① I considered not only the sale of bonds but also the investment in replacement bonds, which improves returns by combining new purchases with the sale of bonds. The analysis showed that under a given nominal coupon of interest and an unchanging yield curve, bond replacement can earn more than holding sale bonds to maturity. ② I examined whether the roll-down effect of selling bonds alone can outweigh the gains from holding bonds to maturity through replacement even in the absence of an increase in the coupon rate of newly purchased bonds. The analysis showed that under the assumption of an unchanging yield curve, repeated bond purchases can exceed hold-to-maturity gains without having to wait for the maturity of the bonds. ③ I examined approaches to bond replacement in more realistic situations where the shape of the yield curve, including the nominal coupon of interest, changes. Given

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22) 19.137 years − 1.753 years − 14.350 years ≒ 3.033 years.

the gains from the sale of the bond resulting from the change in the yield curve, the analysis results showed the minimum coupon rate on the newly purchased bond required for the replacement investment gains to exceed the gains from holding the bond to maturity as well as the maximum length of time that an investor can wait after selling the bond before maturity before needing to purchase a bond with the same coupon rate.

Finally, the future prediction of shifts in the yield curve and changes in the coupon rate is necessary for replacement to be more advantageous than holding to maturity, however, this is not addressed in this study and will be the subject of future research.

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