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## Economic Growth and Land Asset Market

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## 1. Introduction

Thomas Piketty, in his ambitious book “Capital in the Twenty-first Century” (2014) states that the rate of return on capital,  $r$  has exceeded the growth rate of output,  $g$  throughout the long history of capitalistic societies. In the words of Piketty, “This fundamental inequality [ $r > g$ ] will play a crucial role in this book. In a sense, it sums up the overall logic of my conclusion. When the rate of return on capital significantly exceeds the growth rate of the economy, then it logically follows that inherited wealth grows faster than output and income.” (pp.25-26). However, Piketty does not explain why the rate of return on capital should be greater than the growth rate of the economy.

Stiglitz (2015b) asserts that three criticisms are raised against the Piketty analysis. First, once it is recognized that even capitalists consume, and that workers save out of wages (for life-cycle savings), then the neat relationship posited by Piketty for the ever increasing capital income ratio and inequality breaks down.

Secondly, the return to capital should be treated as endogenous. If the increase in wealth represented an increase in “capital,” then the law of diminishing

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\* Former Professor/Matsuyama University, Dr. Economics (Kobe University).

\*\*In a sense, this paper is a revised version of Aono (1976). In this paper we mainly focus on the effects of capital gains on land from the view point of equity and efficiency.

returns would imply that the return to capital should have decreased. Once account is taken of the endogeneity of  $r$ , a more subtle analysis of the determinants of wealth inequality is required.

Thirdly, and most importantly, while both wealth and capital are aggregates, they are distinctly different concepts. Once one recognizes this, it becomes easy to reconcile the stylized facts with conventional theory. The wealth income ratio could be increasing even as the capital income ratio (appropriately measured) is stagnating or decreasing. Much of wealth is not produced assets (“machines”) but land or other ownership claims giving rise to rents.

Taking France as a typical example, Homburg (2014) depicts the Figure, and shows the decomposition of national wealth into capital and land between 1978 and 2012. He points out that the substantial increase in the ratio of wealth to GDP is due to the sharp rise in land values (see Homburg (2014) pp. 7. Fig. 3: Capital and Land in France as multiples of GDP).

These arguments indicate that land has played an important role in the explanation of the growth process. An analysis of the forces giving rise to the increase in land values (capital gains on land) enables us to assess whether such an increase in land prices is likely to continue, and identify policies that might militate against the problems caused by capital gains on land.

The purpose of this paper is to incorporate a land asset market into the aggregative model of economic growth focusing on the effects of capital gains on land. We examine the effects of capital gains on the rate of growth of the variables, the price of land, capital accumulation, the increase in wealth and rising inequality, and wealth-income ratio on the steady state growth path.

For this purpose, the following characteristics of land should be noticed.

(1) Land is durable, and non-reproducible, it is in limited supply, and is essential for production.

(2) In our private-enterprise economy, land is held as an asset, therefore there is a land asset market.

The rest of the paper is organized as follows. Section 2 outlines the basic structure of the model, and derives the dynamic equations of the economy. Section 3 examines the existence and the uniqueness of a positive equilibrium, and the rate of growth of the variables and the price of land on the steady state growth path. We further examine the effects on the steady state capital-labor ratio and land values-labor ratio when the propensity to save out of capital gains on land increases. Section 4 examines whether the increase in wealth increases inequality or not, and examines the wealth-income ratio along the steady state growth path. Section 5 summarizes the main results and discusses possible extensions.

## 2. Model

### 2.1. The basic model

The basic model and notation in this section are as follows.

(a) Technology

Output,  $Y$  is a function of inputs of labor,  $N$ , capital,  $K$ , and land,  $L$ . The production function for this output is twice differential, homogeneous of degree one, in addition, it has the following properties: (a) positive and diminishing marginal productivities of the three factors, (b) impossibility of any product without one of the factors. Technical change is assumed to be land-augmenting and labor-augmenting. The rate of land-augmenting technical progress and the rate of labor-augmenting technical progress are denoted as  $h$  and  $\alpha$ , respectively. The above assumptions are summarized by

$$Y = F(K, N^*, L^*), F_K, F_{N^*}, F_{L^*} > 0, F_{KK}, F_{N^*N^*}, F_{L^*L^*} < 0, \quad (1)$$

$$L^* = L e^{bt}, \quad (2)$$

$$N^* = N e^{at}. \quad (3)$$

(b) The supply of land and labor

Land is assumed to be fixed in supply, and labor is assumed to grow at the given exponential rate,  $n$ . Thus the sum of the rate of labor-augmenting technical progress and the rate of growth of labor is  $\alpha + n$ . We assume that the following conditions are satisfied.

$$L = \bar{L}, \bar{L} : \text{constant} \quad (4)$$

$$N = N_0 e^{nt}, \quad (5)$$

$$0 < \alpha + n < 1. \quad (6)$$

(c) Land asset market

Now consider a portfolio equation which determines the price of land. Although capital and land are indispensable for production, there is a great difference between land and capital. That is, capital is reproducible, less malleable and less alterable to other uses. On the other hand, land is non-reproducible, durable, alterable to other uses and limited in supply. In our private-enterprise economy, land is held as an asset. Therefore, there exists a land asset market.

For analytical simplicity, we assume that there exists a perfect land asset market. Portfolio equilibrium requires that alternative investment options yield the same net rate of return. Since capital is the only asset other than land, and since these two assets have the same risk properties under the assumption of a perfect land asset market, we obtain the following equilibrium condition ;

$$\frac{\dot{P}^e}{P} + \frac{\rho^e}{P} = r^e, \quad (7)$$

where  $P$  is the price of land in terms of goods,  $\dot{P}^e$  is the expected change of land prices,  $\rho^e$  is the expected rent of land, and  $r^e$  is the expected rate of return on capital. The equation (7) tells us that the land asset market is in equilibrium when the expected rate of increase in land prices,  $\dot{P}^e/P$  plus the expected rent-price ratio,  $\rho^e/P$  are equal to the expected rate of return on capital,  $r^e$ .

The equation (7) holds in the following way. When output increases, the expected rent of land increases, and this leads to the expected increase in land prices. The process continues until the expected rate of return on land ( $\dot{P}^e/P + \rho^e/P$ ) is equal to the expected rate of return on capital ( $r^e$ ). It should be noted that when land is indispensable for production, non-reproducible and limited in supply, sooner or later, landowners seek not only rents, but also capital gains on land. The increase in rents comes first, then, capital gains on land come next, not the other way around. If there were no increase in rents, there were no capital gains on land.

For simplicity, we assume that expected and actual price changes are identical, i. e.

$$\frac{\dot{P}^e}{P} = \frac{\dot{P}}{P}, \quad \rho^e = \rho, \quad r^e = r. \quad (8)$$

Two interpretations can be given to the equation (8): first, landowners have a short-run perfect foresight; second, landowners adjust their expectations instantaneously.

In the real world, when individuals expect the future rents and land prices, they do not really know what the future rents and land prices will be like 20 years from now and therefore have little basis for making a judgment about the future rents and land prices. According to psychological research, individuals have a tendency to

favor information that confirms their assumptions, preconceptions or hypotheses whether these are actually and independently true or not. This distortion is called “confirmation bias.”

In shaping collective behavior, these distorted views can influence the situation to which they relate because false views lead to inappropriate actions. This is “the principle of reflexivity” called by George Soros. Soros asserts that “recognizing reflexivity has been sacrificed to the vain pursuit of certainty in human affairs, most notably in economics, and yet, uncertainty is the key feature of human affairs.” (Soros (2009) pp. 4-5). It is well known that Keynes described markets as a beauty contest where the winner is the one who assessed correctly what the other judges would judge to be the most beautiful (Stiglitz (2012) pp. 151).

It is important to consider the implications and the limitations of the equation (7) and (8). We do not assume that landowners have a long-run perfect foresight because they do not know what will happen in the long and distant future. We assume that landowners adjust their expectations instantaneously for analytical simplicity.

(d) Consumption function and savings function

We now consider the consumption function, and hence the savings function. We shall assume that landowners can consume or save out of capital gains on land. We also assume a generalized Cambridge savings function instead of a proportional savings function. Two things should be noticed. First, since profits, wages, rents and capital gains on land are different in character, we assume that the propensity to save out of profits, wages, rents and capital gains on land are not always equal, and that the propensity to save out of profits and rents are not less than the propensity to save out of wages. Second, since capital gains on land are the different characteristic from the other incomes, we assume that the propensity to save out of

capital gains on land is not less than the propensity to save out of profits and rents. When there is an increase in output, landowners are able to increase rents and capital gains on land by virtue of their ownership without doing anything else. In this respect, rents and capital gains on land have the same characteristic in common. But, so far as production is concerned, land is essential for production, therefore, landowners receive annual rents. On the other hand, capital gains on land has nothing to do with production, it is just the increase in wealth of landowners. As we shall explain later, capital gains on land play an important role in the working of our model.

For analytical simplicity, we assume that workers do not own their land, and that they pay the rental rate of land  $\rho$  to landowners.

If landowners save a constant fraction of capital gains on land, then, the consumption function is

$$C = Y - \{s_{\pi}(r + \delta)K + s_W(RN - \rho L) + s_{L1}\rho L\} + (1 - s_{L2})\dot{P}L$$

with  $0 \leq s_W < s_{L1}$ ,  $s_{\pi} \leq s_{L2} < 1$ ,

(9)

where  $s_{\pi}$  : the saving rate of capitalists (entrepreneurs) out of profits,  $s_W$  : the saving rate of workers,  $s_{L1}$  : the saving rate of landowners out of rents,  $s_{L2}$  : the saving rate of landowners out of capital gains on land.  $\delta$  : the rate of depreciated capital.  $r$  : the net rate of return on capital,  $R$  : the real wage rate,  $\rho$  : the rental rate of land.

Since total output,  $Y$  is equal to consumption,  $C$  plus capital depreciation,  $\delta K$ , plus net investment,  $\dot{K}$ , we get

$$Y = C + \delta K + \dot{K}. \tag{10}$$

Substituting (9) into (10) yields

$$\begin{aligned} \dot{K} + \delta K &= \{s_\pi (r + \delta)K + s_W (RN - \rho L) + s_{L1}\rho L\} - (1 - S_{L2})\dot{P}L \\ &\text{with } 0 \leq s_W < s_{L1}, s_\pi \leq s_{L2} < 1, \end{aligned} \quad (11)$$

## (e) Determination of factor prices

Given  $R$ ,  $\rho$  and the production function which is homogeneous of degree one (exhibits constant returns to scale), a competitive producer can maximize the net rate of return on capital when the marginal productivities of capital,  $F_K$ , labor,  $F_N$  and land,  $F_L$  are equal to  $r + \delta$ ,  $R$ , and  $\rho$ , respectively. Thus, we get

$$F_K = r + \delta, \quad (12)$$

$$F_N = R, \quad (13)$$

$$F_L = \rho. \quad (14)$$

## 2.2. Derivation of dynamic equations

It turns out that the dynamics of economy can best be described in terms of the ratios,  $l \equiv PL/N^*$ ,  $u = L^*/N^*$  and  $k \equiv K/N^*$ .

From the equation (1), we obtain

$$\frac{Y}{N^*} = F(K/N^*, 1, L^*/N^*).$$

Thus, letting  $y = Y/N^*$ , the production function can be rewritten as a function of  $k$  and  $u$ .

$$Y = f(k, u). \quad (15)$$

Differentiating the above equation with respect to  $K$ ,  $N^*$  and  $L^*$ , we get

$$F_K = f_k(k, u), \quad (16)$$

$$F_{N^*} = f(k, u) - kf_k(k, u) - uf_u(k, u), \quad (17)$$

$$F_{L^*} = f_u(k, u). \quad (18)$$

For analytical simplicity, we shall assume that  $u$  is constant. Since  $u = L^*/N^*$ , the assumption implies that technical change is of such a character that the rate of land-augmenting technical progress is equal to the sum of the rate of growth of labor supply and the rate of labor-augmenting technical progress. Thus,  $F_K$ ,  $F_{N^*}$  and  $F_{L^*}$  can be written as a function of  $k$ .

Differentiating  $l \equiv PL/N^*$  with respect to time  $t$ , we get

$$\hat{l} = \hat{P} + \hat{L} - \hat{N}^*, \quad (19)$$

where  $\hat{l} = l/l$ ,  $\dot{l} = dl/dt$ . Substituting (7) and (8) into (15), and using (3), (4), (5), (12) and (14), we obtain

$$\hat{l} = -\frac{F_L}{P} + F_K - (\delta + \alpha + n).$$

Using the definition of  $l \equiv PL/N^*$  and substitute (16) and (18) into the above equation, we find

$$\hat{l} = -uf_u + \{f_k - (\delta + \alpha + n)\}l \quad (20)$$

We shall now derive  $\hat{k}(k, l)$ . From the definition of  $k \equiv K/N^*$ , we get

$$\hat{k} = \hat{K} - \hat{N}^* = \hat{K} - (\alpha + n). \quad (21)$$

From (11), we obtain

$$\hat{K} + \delta = \left\{ s_\pi (r + \delta) + s_w R \left( \frac{N}{K} - \rho \frac{L}{K} \right) + s_{L1} \rho \frac{L}{K} \right\} - (1 - s_{L2}) \frac{\dot{P}L}{K}$$

Using (4), (12), (13), (14),  $F_L = F_{L^*} e^{ht}$  and  $F_N = F_{N^*} e^{\alpha t}$ , the above equation can be rewritten as

$$\dot{K} + \delta = s_\pi F_K + s_W \left( \frac{F_{N^*}}{k} - F_{L^*} \frac{u}{k} \right) + s_{L1} F_{L^*} \frac{u}{k} - (1 - s_{L2}) \dot{P}. \quad (22)$$

Also,  $\dot{P}$  can be rewritten as

$$\dot{P} = s_\pi f_k - \frac{u F_L}{l} + F_K - \delta. \quad (23)$$

Substituting (23) into (22) and using (16), (17) and (18), we get

$$\begin{aligned} \dot{K} + \delta &= s_\pi f_k + s_W (f - k f_k - u f_u - u f_u) \frac{1}{k} + s_{L1} f_u \frac{u}{k} \\ &\quad - (1 - s_{L2}) \left( -\frac{u f_u}{l} + f_k - \delta \right) \frac{l}{k}. \end{aligned} \quad (24)$$

If we substitute (24) into (21), we obtain

$$\begin{aligned} \dot{k} &= s_\pi k f_k + s_W (f - k f_k - u f_u - u f_u) + s_{L1} u f_u + (1 - s_{L2}) u f_u \\ &\quad - (1 - s_{L2}) (f_k - \delta) l - (\delta + \alpha + n) k. \end{aligned} \quad (25)$$

The dynamic behavior of the system is thus described by (20) and (25).

### 3. Characteristics of the equilibrium growth path

#### 3.1. The existence and the uniqueness of a positive equilibrium

We first show the existence and the uniqueness of a positive equilibrium of the system,  $(k^*, l^*) > (0, 0)$ . For this purpose, we set  $\dot{l}$  equal to zero in (20) and we have

$$l = \frac{u f_u}{f_k - (\delta + \alpha + n)}. \quad (26)$$

Let  $\tilde{k}$  be defined as the root of

$$f_k - (\delta + \alpha + n) = 0. \quad (27)$$

A finite equilibrium ( $l^* > 0$ ) requires  $k^* > 0$ , and  $f_k > (\delta + \alpha + n)$ , which necessitate  $0 < k^* < \tilde{k}$ .

Substituting (26) into (25), we find

$$\begin{aligned} \Phi(k) &\equiv \dot{k}|_{l=0, l>0} \\ &= s_\pi k f_k + s_W (f - k f_k - u f_u - u f_u) + s_{L1} u f_u - (\delta + \alpha + n) k \\ &\quad - \frac{u f_u (1 - s_{L2}) (\alpha + n)}{f_k - (\delta + \alpha + n)}. \end{aligned} \quad (28)$$

Now define  $0 < \underline{k} < \varepsilon$  for sufficiently small  $\varepsilon$ . Then,

$$\Phi(\underline{k}) > 0, \quad (29)$$

and

$$\lim_{k \rightarrow \tilde{k}} \Phi(k) < 0. \quad (30)$$

As  $\Phi(k)$  is a continuous function in  $k$ , the existence of a root  $k^*$  satisfying  $\Phi(k^*) = 0$ , and  $0 < \underline{k} < k^* < \tilde{k}$  has been proved.

The uniqueness of such a root  $k^*$  is guaranteed if

$$\Phi_k(k)|_{\Phi(k)=0} < 0. \quad (31)$$

Differentiating (28) with respect to  $k$ , we get

$$\begin{aligned} \Phi_k(k) &= s_\pi f_k - (\delta + \alpha + n) + (s_\pi - s_W) k f_{kk} + (s_{L1} - 2s_W) u f_{uk} \\ &\quad - \frac{(1 - s_{L2}) (\alpha + n) [u f_{uk} \{f_k - (\delta + \alpha + n)\} - f_{kk} u f_u]}{\{f_k - (\delta + \alpha + n)\}^2}, \end{aligned} \quad (32)$$

where  $f_{kk}$  and  $f_{uk}$  are the respective second-order partial derivatives of  $f(k)$ .

By the assumption of diminishing marginal productivity,  $f_{kk}$  is negative. Under the further assumption that the factors are cooperative,  $f_{uk}$  is positive. For simplification, we assume that the propensity to save out of capital gains on land,  $s_{L2}$  is not less than the propensity to save out of profits,  $s_{\pi}$  which is equal to the propensity to save out of rents,  $s_{L1}$ . We assume that  $s_{L1}$  is not less than  $2s_W$  ( $s_{L1} \geq 2s_W$ ), where  $s_W$  is the propensity to save out of wages.

From the equation (25), (26) and (28), we see that  $s_{\pi}f_k$  is less than  $\delta + \alpha + n$  at the equilibrium point,  $\Phi_k(k) = 0$ . Consequently, from (32),

$$\Phi_k(k)|_{\Phi(k)=0} < 0$$

with  $0 < 2s_W \leq s_{L1} = s_{\pi} \leq s_{L2} < 1$ .

Thus, the uniqueness of the equilibrium point,  $(k^*, l^*) > (0, 0)$  is proved.

### 3.2. The rate of growth of the variables on the steady growth path

Let us examine now the rate of growth of the variables and the price of land on the steady growth path. Since  $P = lN^*/L = lN_0e^{nt}e^{\alpha t}/L$ , and  $\rho = F_L = F_{L^*}e^{ht}$ ,  $u = L^*/N^*$ , it follows that

$$\hat{P} = \hat{\rho} = \alpha + n = h. \quad (33)$$

Since along the steady growth path,  $\hat{k} = \hat{K} - \hat{N}^* = 0$ , It follows that

$$\hat{K} = \alpha + n. \quad (34)$$

Since  $r = f_k(k) - \delta$ , and  $(r + \delta)K/Y = kf_k(k)/f(k)$ ,

$$\hat{r} = 0, \text{ and } \hat{r} + \hat{K} - \hat{Y} = 0; \text{ hence, it follows that}$$

$$\hat{Y} = \hat{K} = \alpha + n. \quad (35)$$

Since  $R = F_N = F_{N^*}e^{\alpha t}$ , it follows that

$$\dot{R} = \alpha. \quad (36)$$

The above results will be summarized as follows. On the steady growth path, the rates of growth of capital and output are compounded of the rate of growth of the labor force and the rate of labor-augmenting technical progress,  $\alpha + n$ , and the rate of growth of the real-wage rate equals the rate of labor-augmenting technical progress,  $\alpha$ . If land is fixed in supply, the rate of growth of the rent and land prices are equal to the rate of growth of output,  $\alpha + n$ .

Furthermore, from (26), we see that the steady state price of land at time  $t$  is :

$$P(t) = \frac{\rho(t)}{r_* - (\alpha + n)}. \quad (37)$$

It is of sufficient economic interest to pursue the implications of (26) and (37). Equation (26) and (37) tell us that the net rate of return on capital (the rate of profits),  $f_k(k_*) - \delta = r_*$  must exceed the rate of growth of output,  $\alpha + n$ , if the equilibrium price of land is to be positive and finite. It is clear that the above result depends upon the portfolio balance equation. As we stated earlier, asset holders require that the two assets yield the same net rate of return. On the steady growth path, the net rate of return on capital,  $r_*$  must be equal to the rent-price ratio,  $\rho/P$  plus the rate of increase in land prices,  $\dot{P}/P$ . On the other hand, under the assumption of fixed land supply, the rental rate of land,  $\rho$  and land prices,  $P$  grow at the rate of growth of output,  $\alpha + n$ . Thus, the net rate of return on capital  $r_*$  must exceed the rate of growth of output,  $\alpha + n$ , when the equilibrium price of land is to be positive and finite. In other words, the landowner who seeks not only rents, but also capital gains on land makes the net rate of return on capital exceed the rate of growth of output.

Piketty (2014) argues that the rate of return on capital has exceeded the growth

rate of output throughout the long history of capitalistic societies. However, he does not explain why the rate of return on capital should be greater than the growth rate of the economy. In our model, on the steady state growth path, when landowners seek capital gains on land, the net rate of return on capital must exceed the rate of growth of output, but capital-income (output) ratio stays constant.

### 3.3. Comparative Statistics

Now let us examine the effects on the steady state value of  $k$  and  $l$  when the propensity to save out of capital gains on land,  $s_{L2}$  increases.

Setting  $\dot{l} = \dot{k} = 0$  in (20) and (25), and differentiating totally, we get in matrix form

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \frac{dk^*}{ds_{L2}} \\ \frac{dl^*}{ds_{L2}} \end{bmatrix} = \begin{bmatrix} 0 \\ uf_u - (f_k(k) - \delta)l^* \end{bmatrix}, \quad (38)$$

where

$$A = -uf_{uk} + l_*f_{kk}$$

$$B = f_k - (\delta + \alpha + n),$$

$$C = s_\pi f_k - (\delta + \alpha + n) + k_*f_{kk}(s_\pi - s_W) + uf_{uk}(s_{L1} - 2s_W) \\ + (1 - s_{L2})(uf_{uk} - l_*f_{kk})$$

$$D = -(1 - s_{L2})(f_k - \delta).$$

The determinant,  $\Delta$  of the matrix in (38) is :

$$\Delta = (1 - s_{L2})(\alpha + n)(uf_{uk} - l_*f_{kk}) \\ - \{f_k - (\delta + \alpha + n)\} \{s_\pi f_k - (\delta + \alpha + n) + k_*f_{kk}(s_\pi - s_W) \\ + uf_{uk}(s_{L1} - 2s_W)\} > 0 \quad (39)$$

with

$$0 < 2s_W \leq s_{L1} = s_\pi \leq s_{L2} < 1.$$

$\Delta$  is positive if we take (25) and (28), and  $k_* f_{kk} + u f_{uk} = -N^* F_{N^*K}$  into consideration. Using Cramer's rule, we get :

$$\begin{aligned} \frac{dk_*}{ds_{L2}} &= \frac{1}{\Delta} \{ (f_k - \delta) l_* - u f_u \} \{ f_k - (\delta + \alpha + n) \} \\ &= \frac{1}{\Delta} \{ f_k - (\delta + \alpha + n) \} l_* (\alpha + n) > 0. \end{aligned} \quad (40)$$

Similarly, it may be proved that

$$\frac{dl_*}{ds_{L2}} = \frac{1}{\Delta} (u f_{uk} - l_* f_{kk}) l_* (\alpha + n) > 0. \quad (41)$$

From (40) and (41), we see that an increase in the propensity to save out of capital gains on land increases both the steady state capital-labor ratio and the steady state price of land. In other words, when landowners seek capital gains on land and consume out of capital gains on land, they impedes capital accumulation.

Let us consider now the economic implications of the above results. An increase in the propensity to save out of capital gains on land,  $s_{L2}$  means a decrease in the propensity to consume out of capital gains on land, and this leads to a larger proportion of the output is directed into the savings. Since we assume that savings are directed into investment (the increase in the stock of capital), this causes the increase in the stock of capital (see the equation (9) and (10)). Since the price of land grows at  $\alpha + n$  on the steady state growth path, this effect is intensified by a higher land price, because a higher land price leads to a larger capital gains on land when the price of land grows at  $\alpha + n$ .

The increase in capital increases output, and in doing so, causes a rise in rents,  $\rho$  and decreases the marginal productivity of capital,  $F_k$ . Both a rise in  $\rho$

and a decrease in  $F_k$  cause the increase in land prices. Thus an increase in the propensity to save out of capital gains on land causes the increase in land prices.

Since we assume landowners seek capital gains on land, if they increase the propensity to save out of capital gains on land,  $s_{L2}$ , they can raise land prices, and doing so, they can increase capital gains on land. Landowners who hold a lot of wealth in terms of the land value may want to pass significant amounts of wealth across generations. Since in most countries unrealized capital gains on land are not taxed, an increase in capital gains due to the increase in land prices is transferred to the next generation.

On the other hand, if landowners decrease the propensity to save out of capital gains on land,  $s_{L2}$  (increase the propensity to consume out of capital gains on land), they can reduce the steady state price of land. But in this case, as long as landowners seek capital gains on land, an increase in the propensity to consume out of capital gains on land decrease the steady state capital-labor ratio, and in doing so, impedes capital accumulation further more.

#### 4. Effects of the increase in wealth on inequality and wealth-income ratio

##### 4.1. Increase in wealth and rising inequality

Let us examine whether the increase in wealth increases inequality or not. For this purpose, we shall examine the share of workers' real wages to wealth owned by capitalists and landowners, wealth per capita, and the wealth-income ratio on the steady growth path.

For simplicity, we assume that wealth consists of two components, capital and land. The assumption is not so unrealistic as it might seem. Stiglitz (2015a) asserts that "The most important source of the disparity between the growth of wealth and the growth of productive capital is land: much of the increase in wealth is an

increase in the value of land—not associated with any increase in the amount of land” (pp. 2). Homburg (2014) criticizes Piketty’ book [2014] and argues that “Piketty treats the term capital and wealth interchangeably, and deliberately so (pp. 47), because he believes that distinguishing between produced capital and non-produced land is cumbersome. . . . Taking France as a typical example, fig. 3 shows that the decomposition of national wealth into capital and land between 1978 and 2012. . . . The crucial point is that the strong increase in the wealth-income ratio, which commenced in 1999, the year of the introduction of the euro, was driven by an increase in land values, which almost tripled by 2012.” (pp. 7-8)

Let  $K(t)$  denote the stock of produced capital owned by capitalists at time  $t$ ,  $L$  the stock of land fixed in supply owned by landowners, and  $P(t)$  the price of land at time  $t$ , and  $W(t)$  wealth owned by capitalists and landowners at time  $t$ . In every period  $t$ ,  $W(t)$  is given by

$$W(t) = K(t) + P(t)L. \quad (42)$$

Increases in wealth can be due to either capital accumulation or an increase in land prices. Let  $R(t)$  denote real wages per capita at time  $t$ ,  $N(t)$  the numbers of workers at time  $t$ . The share of workers’ real wages to wealth owned by capitalists and landowners at time  $t$ ,  $\beta(t)$  is given by

$$\beta(t) = \frac{R(t)N(t)}{K(t) + P(t)L}. \quad (43)$$

On the steady growth path,  $R$  grows at  $\alpha$ ,  $N$  grows at  $n$ ,  $K$  grows at  $\alpha + n$ ,  $P$  grows at  $\alpha + n$ . Hence,  $\beta$  stays constant. We assume that capital is mostly owned by capitalists, and land is mostly owned by landowners. It would be realistic to suppose that the increase in the numbers of workers is larger than the increase in the numbers of capitalists and landowners. For simplicity, we assume that the numbers

of workers grow at  $n$ , on the other hand, the numbers of capitalists and landowners stay constant. Then, real wages per capita grows at  $\alpha$ , wealth owned by capitalists and landowners per capita grows at  $\alpha + n$ . There exists rising inequality between real wages per capita and wealth per capita. Besides, in the case where landowners seek capital gains on land, the price of land at time  $t$  is determined by  $\rho(t)/\{r - (\alpha + n)\}$ . Thus, when landowners seek capital gains on land, there is much larger rising inequality due to rising land prices. It should be noticed that wealth, especially, land owned by landowners is bequeathed to the heirs in many cases. Thus wealth is transferred to the next generation and the disparity between who own a lot of wealth and who do not own wealth increases.

#### 4.2. Wealth-income ratio on the steady growth path

Letting  $w$  denote wealth per capita, and considering (26), we get

$$\frac{dw}{dk} = \frac{d}{dk}(k+l) = 1 + \frac{uf_{uk}\{f_k - (\delta + \alpha + n)\} - f_{kk}uf_u}{\{f_k - (\delta + \alpha + n)\}^2} > 1. \quad (44)$$

Whether landowners seek capital gains on land or not,  $w$  increases more than  $k$ . Furthermore, the above equation tells us that when they seek capital gains on land,  $w$  increases much more than  $k$ . This is because when landowners seek capital gains on land, the denominator,  $\{f_k - (\delta + \alpha + n)\}^2$  of the right hand side of (44) becomes smaller and dominant.

Let us examine the wealth-income ratio along the steady growth path.

Denoting  $\gamma$  as the wealth-income ratio,  $\gamma$  is given by

$$\gamma = \frac{W}{Y} = \frac{K+PL}{Y} = \frac{w}{f(k)} = \frac{k+l}{f(k)}. \quad (45)$$

Differentiating (45) with respect to  $k$ , we obtain

$$\frac{d\gamma}{dk} = \frac{d}{dk} \left( \frac{k+l}{f(k)} \right) = \frac{\frac{dw}{dk} f(k) - f_k (k+l)}{f(k)^2}. \quad (46)$$

Since  $dw/dk > 1$ , as long as  $Y \cong (r + \delta)W$ ,  $\gamma$  increases more than  $k$ .

Furthermore, from (44), when landowners seek capital gains on land,  $w$  increases much more than  $k$ . Therefore, when landowners seek capital gains on land,  $\gamma$  increases much more than  $k$ .

Now we can ask what happens if the propensity to save out of capital gains on land,  $s_{L2}$  increases. In section 2.3, we have shown that an increase in the propensity to save out of capital gains on land increases both the steady state capital-labor ratio and the steady state price of land. From (44) and (46), it is easily seen that an increase in the propensity to save out of capital gains on land,  $s_{L2}$  causes to increase wealth per capita,  $w$  much more than the capital-labor ratio,  $k$  and causes to increase the wealth-income ratio,  $\gamma$  much more than the capital-labor ratio,  $k$  on the steady state growth path.

The point is the plausibility of an increase in the propensity to save out of capital gains on land in the long run. There are two plausible reasons. First, given the steady state growth rate of output (the rate of increase in land prices), landowners are able to earn larger capital gains on land held for longer periods of time. In addition to capital gains on land, landowners earn annual rents every year. Hence landowners who hold land for longer periods of time are able to consume out of rents and save a larger proportion of capital gains. Second, in most advanced countries realized capital gains on land are taxed, but unrealized capital gains are not taxed. If landowners do not sell their land, the capital gains earned during the landowner's lifetime go entirely untaxed. Because of the present capital gains taxation system, landowners are willing to bequeath their land to their heirs. Thus a large amount of capital gains due to the increase in land prices is transferred to the

next generation.

If the propensity to save out of capital gains on land increases in the long run, the disparity of wealth per capita and the wealth-income ratio will increase in the long run.

## 5. Concluding Remarks

In this paper, we integrated a land asset market into the economic growth model. We have shown that when landowners seek capital gains on land and consume out of the capital gains, the following results accrue on the steady growth path. First, the net rate of return on capital (the rate of profits) must exceed the rate of growth of output, but capital-income (output) ratio stays constant. Second, consumption out of capital gains on land impedes capital accumulation. An increase in the propensity to consume out of capital gains on land decrease the steady state capital-labor ratio, and in doing so, impedes capital accumulation further more. On the other hand, a decrease in the propensity to consume out of capital gains on land increases the steady state capital-labor ratio, and mitigates the negative effect on capital accumulation, but, it raises the steady state price of land.

Third, when landowners seek capital gains on land, there exists larger rising inequality due to rising land prices, and the wealth-income ratio increases much more than capital-labor ratio.

Two extensions of our model are being considered. First, we have examined the characteristics of the steady state growth path in this paper.

Examining the stability of the equilibrium path, and considering the paths which do not converge to the long-run steady state in greater detail are left for future research. Second, we disregard the effects of the capital gains tax on the steady state price of land, capital accumulation and the stability of the system. The effects of the capital gains tax on a capital gains-seeking economy are an interesting

research topic for the future.

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